The Washington State Board of Education

Governance I Achievement I Transitions I Math & Science I Effective Workforce

Title:	**Review of Certificate of Academic Achievement Options for End of Course Exams**
As Related To:	☐ Goal One: Advocate for effective and accountable P-13 governance in public education ☒ Goal Two: Provide policy leadership for closing the academic achievement gap ☐ Goal Three: Provide policy leadership to strengthen students' transitions within the P-13 system ☒ Goal Four: Promote effective strategies to make Washington's students nationally and internationally competitive in math and science ☐ Goal Five: Advocate for policies to develop the most highly effective K-12 teacher and leader workforce in the nation ☐ Other
Relevant To Board Roles:	☐ Policy Leadership ☒ System Oversight ☒ Advocacy ☐ Communication ☐ Convening and Facilitating
Policy Considerations / Key Questions:	Increasing numbers of students projected to access Certificate of Academic Achievement (CAA) options, and the associate cost of providing the options, could lead to a consideration of policy change by the 2013 Legislature.
Possible Board Action:	☒ Review ☐ Adopt ☐ Approve ☐ Other
Materials Included in Packet:	☒ Memo ☐ Graphs / Graphics ☐ Third-Party Materials ☒ PowerPoint
Synopsis:	The graduating classes of 2013 and 2014 will be required to pass one mathematics End of Course (EOC) exam; the graduating class of 2015 and beyond will need to pass two mathematics EOCs and one biology EOC. With these new requirements, more students are likely to participate in the approved alternative assessment options: 1) alternative assessments (ACT/SAT/approved subject AP tests); 2) grade comparisons; and, 3) Collections of Evidence (COE). Of these, COEs are likely to draw the most participants. Staff will provide a preliminary projection through 2015 of the numbers of COEs and an estimate of their cost.

Review of Certificate of Academic Achievement (CAA) Options for End of Course Exams

Policy Consideration

Washington State did not require students in the class of 2012 and prior to pass a state mathematics exam or science exam for graduation. RCW 28A.655.066 (2) adds additional requirements to graduating classes in 2013 and 2014: students in these classes will need to pass one mathematics End of Course Exam (EOC) as a graduation requirement. Starting with the class of 2015, students will need to pass two mathematics EOCs (RCW 28A.655.066 (3)) and one science EOC to graduate (28A.655.061 (4)).

As EOCs become part of graduation requirements, large increases in the number of students accessing approved alternatives to state assessments are projected.

The cost of providing alternative assessment options, particularly Collections of Evidence (COEs), to increasing numbers of students could prompt a consideration of policy change by the 2013 Legislature.

Summary

Districts award a Certificate of Academic Achievement (CAA), or Certificate of Individual Achievement (CIA) for students with an Individualized Education Program, to students who pass the state assessments required for graduation. The state has approved alternatives to state assessments, allowing students options for earning their CAA or CIA. Approved alternatives (CAA Options) are shown in the table below.

CAA Options	
Collection of Evidence	• An evaluation of a set of work samples based on classroom work prepared by the student with instructional support from a teacher
Qualifying Score on an Approved Test	• ACT • SAT • Approved subjects in an Advanced Placement (AP) test
Grade Comparison	• A student's grades in a subject are compared to the grades of other students who took the same course and passed the state exam in that subject • The comparison is conducted by school district personnel • This option is only available to 12th graders with a grade point average of 3.2 or above

Legislation postponed required mathematics and science assessments in 2007 and again in 2011 (see the Background section below), so alternative assessments in mathematics and science have never been fully implemented.

The table below shows the numbers of SAT, ACT and AP scores, and grade comparisons submitted as approved alternative assessments in 2011-2012.

CAA Option Program	Approved by Content Area	Percent of Students who Took the Assessment
ACT, SAT, AP Tests		
Math Approved	738	1.1%
Reading Approved	1,098	1.4%
Writing Approved	785	1.0%
Grade Comparison		
Math Approved	207	0.2%
Reading Approved	11	0.01%
Writing Approved	7	0.01%

It is likely that the number of grade comparisons in mathematics will increase in 2012-2013, since seniors will need to pass the mathematics assessment to graduate. This could cause some extra demands on districts' staff time, since district staff performs the grade comparison.

Overall, the percent of students participating in these CAA options are small. The addition of the EOCs in mathematics and science as graduation requirements may result in a significant increase in the number of students participating in these options, but it is likely to remain a small percent of total students who take the assessments.

Of the CAA Options, Collections of Evidence (COE) are the most numerous for reading and writing, and are likely to be very numerous for mathematics and science.

Students submitted mathematics COEs in 2009 as an alternative to the high school mathematics High School Proficiency Exam (HSPE). Because the COE for the HSPE was for a comprehensive test that was not required for graduation, it is not directly comparable to the COEs students will be attempting in 2013. However, the number of students who participated in this option for mathematics in 2009 may be an indication of how many students will use COEs as an option in 2013. In 2009, 76,576 students attempted the 10^{th} grade mathematics HSPE; 9,448 COEs were scored in mathematics or 12.3 percent of the total number of student who took the mathematics assessment.

The attached chart illustrates the number of collections of evidence from 2009 to 2011, and projects numbers (based on the assumptions listed below the chart) for 2012 to 2015. This chart is a preliminary projection for the purposes of discussion only.

The current budget allots $400 per collection, with $200 going to the district, and $200 going to OSPI to fund the contract for grading. The attached preliminary projection chart shows approximately 28,728 mathematics and science COEs in 2015. With the current budget cost, this represents about $11.5 million in additional cost for COEs in mathematics and science, or $13.6 million for all the subject areas.

Demographic data on student participants in reading and writing COEs suggest that these COEs serve under-represented student populations at a significantly higher rate than the general student population. In February 2011, students participating in reading and writing COEs were 65.7 and 70.3 percent low income students respectively, compared to 43.7 percent low income in the general student populations. Hispanic students, black students, and bilingual/English language-learners also participated in COEs at a disproportionally high rate.

Background

SBE is authorized by RCW 28A.230.090 to set high school graduation requirements, including the certificate of academic achievement and certificate of individual achievement (RCW 28A.230.090 (1)(b)).

The Superintendent of Public Instruction is required to consult with the SBE on the assessment system (RCW 28A.655.070(3)(a)):

> In consultation with the state board of education, the superintendent of public instruction shall maintain and continue to develop and revise a statewide academic assessment system in the content areas of reading, writing, mathematics, and science for use in the elementary, middle, and high school years designed to determine if each student has mastered the essential academic learning requirements identified in subsection (1) of this section. School districts shall administer the assessments under guidelines adopted by the superintendent of public instruction. The academic assessment system may include a variety of assessment methods, including criterion-referenced and performance-based measures.

It is also the responsibility of SBE to identify scores students must achieve to meet the standard on statewide student assessments for high school students to obtain a certificate of academic achievement (28A.305.130 (4)(b)). Cut scores for COEs in reading and writing were approved by SBE in April 2008. The Board will be asked to approve the standard setting process and cut scores for alternative assessments to the mathematic EOCs at the November 2012 Board meeting.

Legislation postponed implementation of mathematics and science assessments as a graduation requirement with ESSB 6023 in 2007, and again in 2011, with HB 1412 and ESHB 1410. Assessments in mathematics required for graduation for the graduating classes of 2013, 2014, and 2015 are specified in RCW 28A.655.066(2) and (3). Assessments in science required for graduation for the graduating class of 2015 is specified in RCW 28A.655.06 (4).

Video of the Senate floor debate on ESSB 6023 is available on TVW at http://tvw.org/index.php?option=com_tvwplayer&eventID=2007040142B , at 1.06.50 on the timer and video of the House of Representatives floor debate on ESHB1410 is available at http://tvw.org/index.php?option=com_tvwplayer&eventID=2011050127B , at 29.30 on the time.

The table below highlights some of the legislation establishing the current required assessments and alternative assessments.

Legislation	Year	Highlights
3ESHB 2195	2004	• Established the Certificate of Academic Achievement • Made the CAA (or CIA) a graduation requirement for the class of 2008, including mathematics assessments • Made science assessment a graduation requirement for the class of 2010 • Directed OSPI to develop alternative assessments
ESSB 6475	2006	• Directed OSPI to implement alternative assessment methods o Grade Comparison o Collection of work o CTE Collection of work o PSAT,SAT, ACT comparison
ESSB 6023	2007	• Students can graduate without a CAA by taking one math credit after the 11th grade for the class of 2008, and two math credits after the 10th grade for the classes of 2009 to 2012; mathematics assessment as a graduation requirement moved to class of 2013 • Moved science assessment from 2010 to 2013 as a requirement • Set GPA requirement of 3.2 as a student eligibility requirement for the grade comparison alternative assessment
ESHB3166	2008	• Directed OSPI to develop statewide EOCs for high school math • Established EOCs as a requirement for the class of 2013
2ESHB 1087 section 513 (budget bill)	2011	• Mandated that a student may submit only one collection of work per content area
HB1412	2011	• Students in the graduating classes of 2013 and 2014 must meet the state standard on one high school math EOC rather than two
ESHB1410	2011	• Students must meet the state standard on the science assessment as a requirement of the class of 2015 rather an the class of 2013 • The science assessment will be a biology EOC • AP exams added to the list of approved alternative assessments

Washington State is one of nine states that require an EOC as a graduation requirement. Of these, four have programs similar to Collections of Evidence. The table below lists the other states and their required EOC assessments and alternatives assessments.

State	End of Course Exams Currently required for Graduation	Options for Students Who Do Not Pass	Source
ARKANSAS	English II Algebra I	Remediation and multiple retakes Alternative assessments: ACT, SAT, IB	Arkansas Rule
FLORIDA	Algebra I Geometry Biology I	Multiple retakes Alternative assessments: ACT, SAT	Florida Comprehensive Assessment Test FAQ Graduation

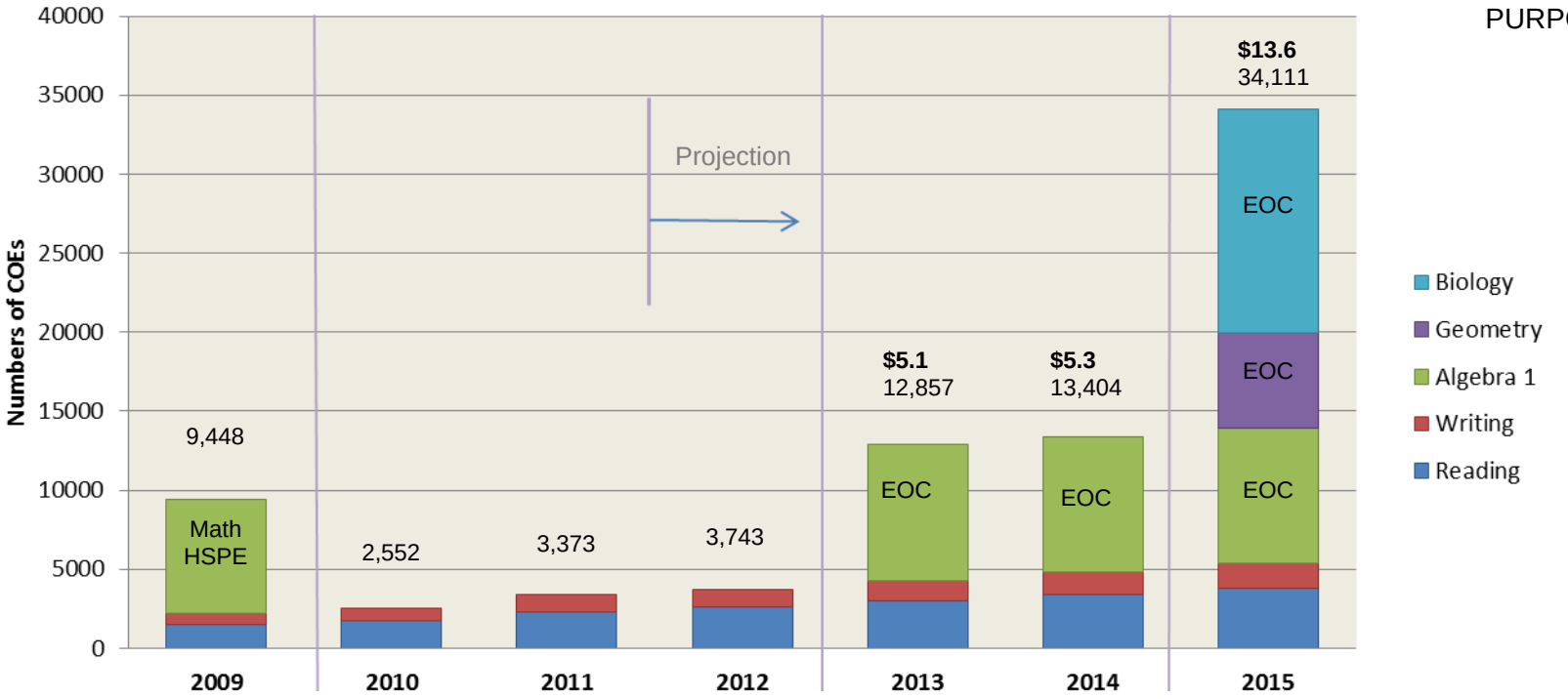
			Requirements for Florida's Statewide Assessments
INDIANA	English 10 Algebra I	Demonstrate mastery of standards by evidence waiver or work-readiness waiver (to be eligible, must retake, GPA, attendance, teacher/administrator recommendations)	Indiana Department of Education website
MARYLAND	English Algebra/Data Analysis Biology Government	Multiple retakes Bridge Plan (eligibility requirements plus project evidence)	Maryland High School Assessments website
MASSACHUSETTES	Pass one of: Biology Introductory Physics Chemistry Technology/Engineering	MA Comprehensive Assessment System (MCAS) Portfolio Appeal consisting of the student's current or cumulative work in a content area	MCAS Performance Appeals website
MISSISSIPPI	English II Algebra I Biology I US History from 1877	Multiple retakes	Mississippi Subject Area Testing Program Second Edition Student/Parent Information Guide
NEW YORK	English Mathematics (Integrated Algebra, Geometry, OR Algebra 2) Science (choice of several) US History and Government Global History and Geography	Alternative assessments: AP, SAT, IB	Part 100 Regulations 100.5 Diploma Requirements
OKLAHOMA	English II Algebra I Two of: English III Algebra II Geometry Biology US History	Multiple retakes Demonstrate mastery of subject matter through an end of course project	Oklahoma School Testing Program FAQ

Action

No action is required at this time

Number of Collections of Evidence (COE), with Projected Estimated Cost in Millions of Dollars

PRELIMINARY PROJECTION FOR DISCUSSION PURPOSES



Assessment Required for Graduation	2009	2010	2011	2012	2013	2014	2015
	HSPE in: -Writing -Reading (COEs were available for math, although math was not required for graduation)	**HSPE in:** -Writing -Reading (ESSB 5889 allows COE only to be submitted in content area required for graduation)			**HSPE in:** -Writing -Reading **EOC in:** -Algebra 1 or Geometry		**HSPE in:** -Writing -Reading **EOC in:** -Algebra 1 -Geometry -Science **SBAC Next-Generation Assessments?**

Assumptions:

1. The total number of students taking assessments will be approximately 80,000 per year.
2. The percent that do not meet standard and chose to take the COE will be 30 percent for math, 60 percent for science.
3. Students will attempt COEs in their senior year.
4. The number of Reading and Writing COEs will continue to increase linearly through this period.
5. The cost per COE will be $400.
6. The pass rates for math and science used for this estimate is the pass rate for the first year of the Algebra 1, Geometry, and Biology End-of-Course (EOC) assessments.
7. The pass rate used for Reading and Writing is the average pass rate for the Reading and Writing HSPE from 2009 to 2011.

EXAMPLE OF MATHEMATICS COLLECTION OF EVIDENCE

The Following 26 Pages Includes Student Work from
Mathematics Collection of Evidence Task Development

- Each student collection consists of 6 to 8 work samples
- Each work sample is related to Performance Expectations of the Washington State Mathematics Standards
- The Inclusion Tasks were submitted by teachers throughout the state, have undergone a peer review process, and are edited to ensure alignment with standards.

For Planning Purposes Only – Not to be included in Student Collections

Algebra Work Sample Documentation Planning Form

To be scored, the collection must include:

- ❑ a minimum of six (6) and a maximum of eight (8) work samples
- ❑ at least two (2) different demonstrations of all four strands across the whole breadth and depth of the collection
- ❑ at least two (2) on-demand work samples (from two different strands)
- ❑ two (2) examples across the collection of process PEs to meet sufficiency purposes only
- ❑ two (2) examples across the collection of content-specific course material to sufficiency purposes only
- ❑ the entire task, directions or questions that accompany each work sample

The work sample must include:

- ❑ work that encompasses the entire performance expectation

Title of Algebra Task	Reporting Strands				Sufficiency Only		On Demand
	Numbers, operations, expression and variables	Linear equations and inequalities	Characteristics and behaviors of linear and non-linear functions	Data and statistics	Process (Sufficiency only)	Course Specific Content (sufficiency only)	
1. Building a House		A1.4.B	A1.3.C				❑
2. How Close is Lightning		A1.1.B	A1.3.C	A1.6.D			❑
3. Pre Season Football		A1.1.C		A1.6.B	A1.8.A		❑
4. Skate Park	A1.2.B				A1.8.A, B	A1.5.A	❑
5. Goat Circles	A1.7.D				A1.8.C, E		❑
6. Safe Driving	A1.7.D	A1.1.C	A1.3.B			A1.1.D	❑
7.							❑
8.							❑

Building a House

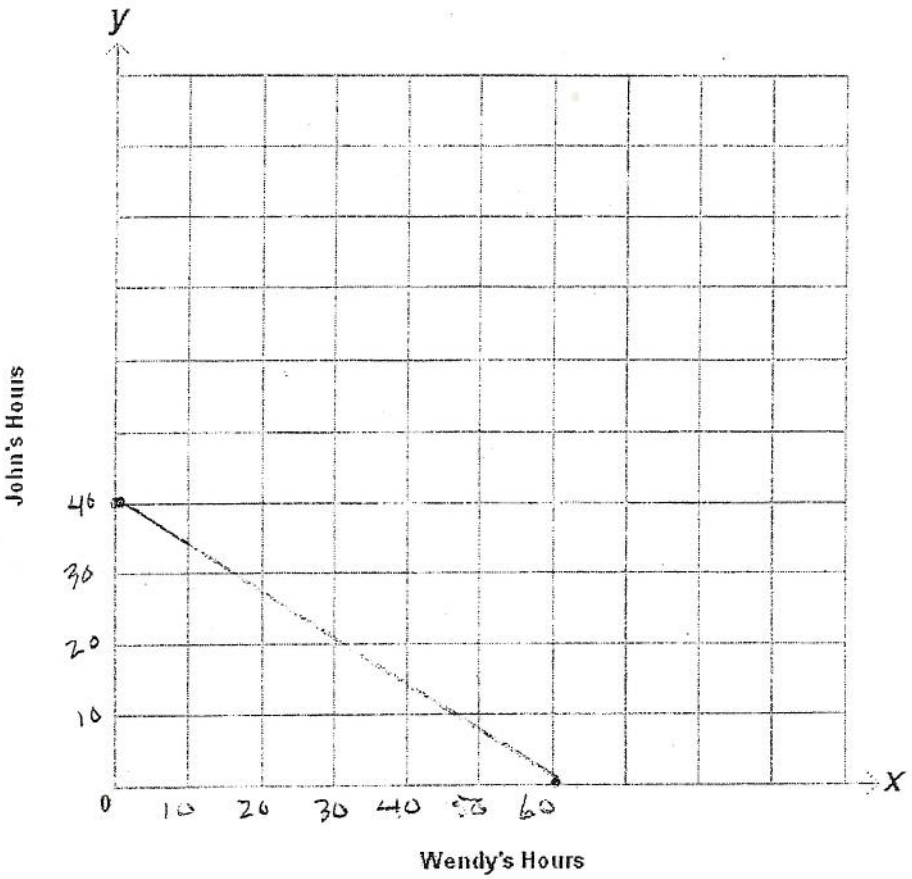
Don is two weeks away from finishing building a house and needs to hire one or more workers. He reads through the local paper and finds two people, Wendy and John, who are advertising for construction work. Wendy has construction experience and charges $15 per hour. John has less experience than Wendy and is charging $10 per hour.

(A1.4.B)(M1.3.D)

1a. Don wants to graph the combinations of the number of hours he can hire Wendy compared to the number of hours he can hire John. Don has budgeted $600 to hire workers. Don realizes, at the extremes, he could hire only Wendy to work for 40 hours or hire only John to work for 60 hours. Don writes the ordered pairs (40,0) and (0,60) to represent these two extremes, where the x-value represents the number of hours Wendy works and the y-value represents the number of hours John works.

Graph the line that passes through the points (40,0) and (0,60).

Be sure to include scales for both axes.



Inclusion Task

1b. Write an equation in slope-intercept form that represents a line that passes through the points (40,0) and (0,60).

Equation:

$y = mx + b$ $\quad y = \frac{3}{2}x + b$

Slope = $\frac{\text{rise}}{\text{run}} = \frac{+60}{-40} = \frac{+3}{-2}$

1c. Rewrite the equation from slope-intercept form to standard form.

Show the algebraic steps you took to rewrite the equation.

$+\frac{3}{2}x - y = \frac{3}{2}x + b$

$-\frac{3}{2}x$

$+\frac{3}{2}x + y = b$

Equation:

$\frac{3}{2}x + y = b$

Inclusion Task

(A1.3.C)(M1.2.C)

2a. Don decides to hire both John and Wendy for two weeks of work. Don uses the function $f(x) = -\frac{2}{3}x + 40$, where x is the number of hours John works and $f(x)$ is the number of hours Wendy works, to determine the relationship between how many hours each person works. Don plans to have John work 43.5 hours during the two weeks.

Evaluate $f(43.5)$.

Show how you algebraically solved the problem.

$f(x) = -\frac{2}{3}x + 40$

$f(43.5) = -\frac{2}{3}(43.5) + 40$

$f(43.5) = 69$

$f(43.5) = 69$

2b. Explain what the value of $f(43.5)$ represent?

f(x) is Wendy hours. The equation f(x) = -2/3 + 40 says x = Johns hour and f(x) = Wendys. when x = 43.5 (Don's hour) then by solving I get the Wendy's hours. f(x)

Inclusion Task

2c. Don uses the function $f(x) = -\frac{2}{3}x + 40$, where x is the number of hours John works and $f(x)$ is the number of hours Wendy works, to determine the relationship between how many hours each person works. At the end of two weeks, he finds that Wendy has worked 26 hours.

Solve for $f(x) = 26$.

Show how you algebraically solved the problem.

$f(x) = -\frac{2}{3}x + 40$

$26 = -\frac{2}{3}x + 40$

$-40 \quad -40$

-14

$-14 = -\frac{2}{3}x$

$+\frac{3}{2}(-14) = -\frac{2}{2}(\frac{x}{3})x$

$21 = x$

$x = 21$

2d. What does the solution to $f(x) = 26$ represent?

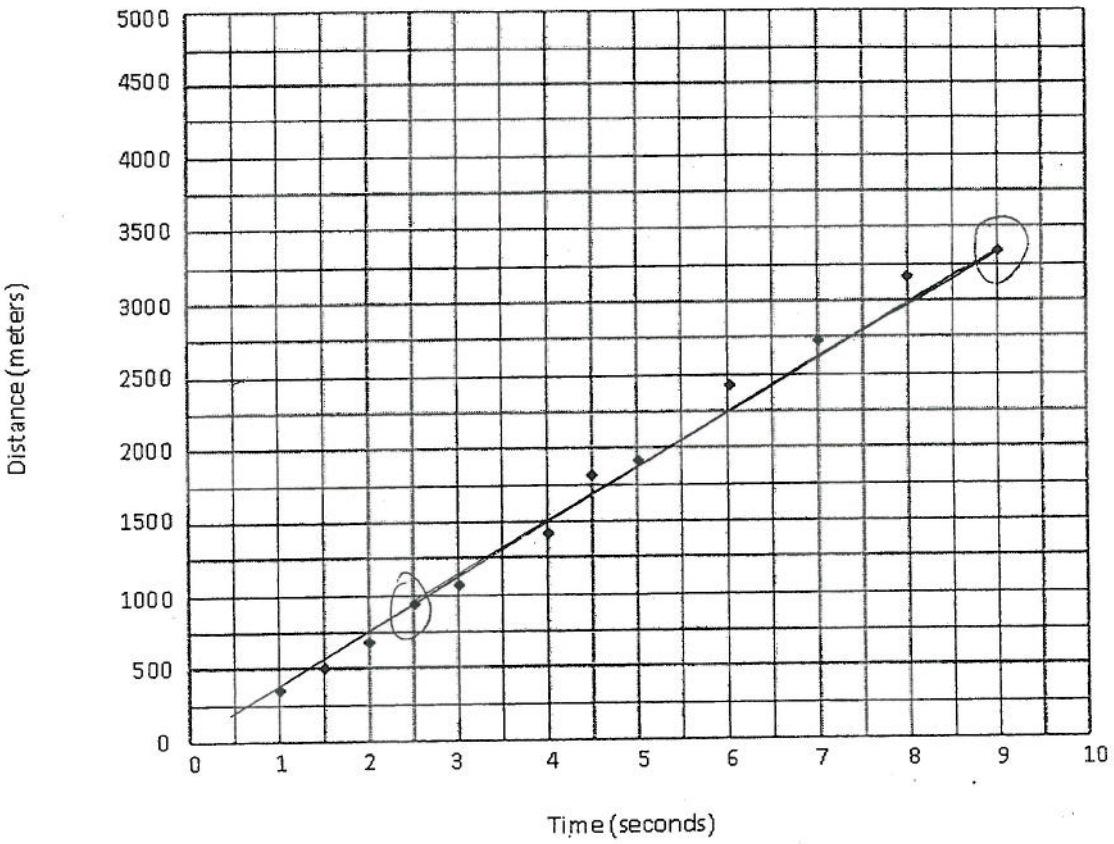
$f(x) = 26$ is the hour wendy works. when I solve for x I should get 21 hours

Inclusion Task

How Close is Lightning?

During a thunder storm, Cindy uses a stopwatch to time (in seconds) from when she sees a flash of lightning to when she hears the crash of thunder. The next day, she uses a map and a ruler to determine the distances to the lightning strikes. The time between the lightning and the thunder, in seconds, and the distance from Cindy, in meters, are graphed in the scatterplot.

How Close is Lightning?



1a. Use a ruler and draw a line that fits the data in the scatterplot.

(2.5, 1000) (9, 3200)

$\frac{3200 - 1000}{9 - 2.5} = \frac{2200}{6.5} = 338.5$

$y = 338.5x + 153.75$

$1000 = 338.5(2.5) + b$

$1000 = 846.25 + b$

$-846.25 \quad -846.25$

$153.75 = b$

DRAFT

(A1.6.D)

2a. What is an equation for a line that fits the data in the scatterplot? $y = 338.5x + 153.75$

y = Distance in Meters
x = seconds

Be sure to define any variables you use.

Equation: $y = 338.5x + 153.75$

Variables: y = Distance in meters

x = seconds

2b. Describe what the slope of the line represents in the situation.

how many meters the lightning travels a second.

2c. Describe what the y-intercept of the line represents in the situation.

y = the distance the lightning travel in "x" amount of seconds

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2d. The next night, there was another thunderstorm. Cindy saw lightning strike the top of Cougar Mountain which she knows is approximately 4,500 meters away from her. She, however, did not have a stopwatch to time from when she saw the lighting to when she heard the thunder.

Using the equation you wrote, predict how many seconds would pass between the lightning and thunder when the distance to a lightning strike is 4,500 meters.

Show the steps you used to solve the equation.

$4500 = 338.5x + 153.75$

$-153.75 \quad -153.75$

$\frac{4346.25}{338.5} = \frac{338.5x}{338.5}$

$2.84 = x$

seconds

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2e. Cindy runs to get her stopwatch so she can time from when she sees the lightning strike to when she hears the thunder. On the next lightning strike she sees, she times 14 seconds until she hears the thunder.

Using the equation you wrote, predict the distance to a lightning strike when the time between the lightning and the thunder is 14 seconds.

Show the steps you used to solve the equation.

$y = 338.5(14) + 153.75$

$y = 5439 + 153.75$

$y = 5592.7$ meters

In dry, 68° F air, sound travels approximately 343.2 meters per second. The equation $f(t) = 343.2t$ where $f(t)$ is the distance, in meters, and t is time, in seconds can be used to model the relationship between distance and time for the speed of sound.

(A1.3.C)

3a. Evaluate $f(14)$.

Show your work algebraically.

$f(14) = 343.2(14)$

$f(14) = 4804.8$ meters

3b. What does the value of $f(14)$ represent?

The number of seconds.

DRAFT

3c. Solve $f(t) = 4{,}500$.

Show your work algebraically.

$\frac{4500}{343.2} = \frac{343.2t}{343.2}$

$13.1 = t$

seconds

3d. What does the solution of $f(t) = 4{,}500$ represent?

$f(t) = 4500$ is the distance in meters that the sound traveled.

DRAFT

Ueshae Hamson

□ On-demand | Pre-Season Football | *(A1.1.C, A1.6.B)*

Pre-Season Football

Bob and Jim are getting ready for football season. Bob wants to lose weight but Jim wants to gain weight. Trainer Sue weighed both players and developed weight loss and gain plans for each player.

Bob
Starting weight: 304 lbs
Weight loss: 1.25 lbs per week

Jim
Starting weight: 150 lbs
Weight gain: 0.75 lbs per week

(A1.1.C)

1a. Bob and Jim start Trainer Sue's weight plan at the same time.

When Bob loses and Jim gains weight according to Trainer Sue's plan, after how many weeks will Bob and Jim weigh the same amount?

- Write a system of equations that represents Bob's and Jim's weight each week if they follow Trainer Sue's plan. Be sure to define the variables you use.
- Show the steps you used to solve the system of equations.

System of Equations: Substitution

Variables: X = weight

y = amount

Bob = $y = 304x - 1.25$

Jim = $y = 150x + .75$

$y = 150(.012) + .75$

$y = 2.55$

$304x - 1.25 = 150x + .75$

$+1.25 \quad +1.25$

$304x = 150x + 2$

$-150 \quad -150$

$\frac{154x}{154} = \frac{2}{154}$

$x = .012$

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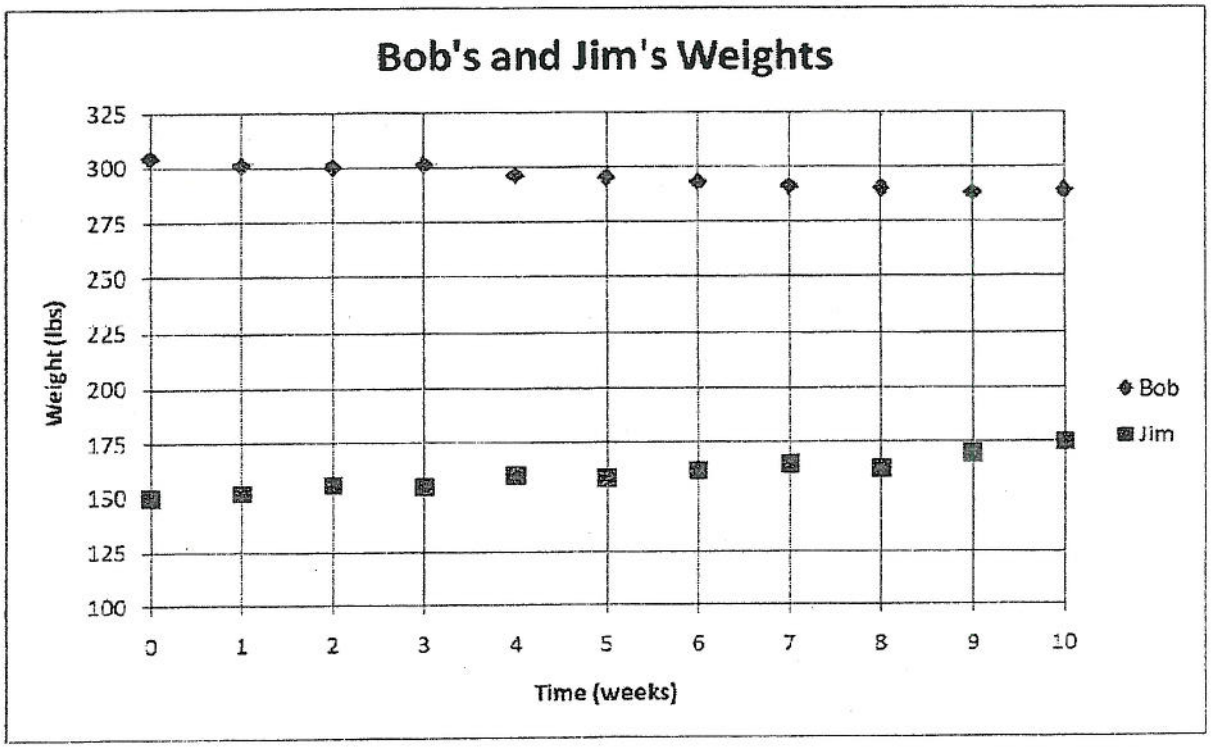
The scatter plot shows Bob's and Jim's actual weight during the first 10 weeks.



(A1.6.B)

2a. Based on the data, which football player better followed Trainer Sue's weight plan?

Use specific data from the scatter plot and Trainer Sue's plan to support your answer.

The football player who followed Trainer Sue's Plan was Bob because the data shows him constantly loosing weight and not gaining it.

DRAFT

Name [redacted]

Skatepark

HomeLink G

At the Rotary Skatepark, Taryn and Jaden are practicing their ramp skills. At the top of the ramp, their potential energy is given by the formula $P = 9.8mh$ where P is potential energy in Joules, m is mass in kilograms, and h is height in meters. At the bottom of the ramp, their kinetic energy is given by the formula $K = \frac{1}{2}mv^2$ where K is kinetic energy in Joules, m is mass in kilograms, and v is velocity in meters per second.

(A1.5.A)

1a. Taryn's mass is 52 kilograms.

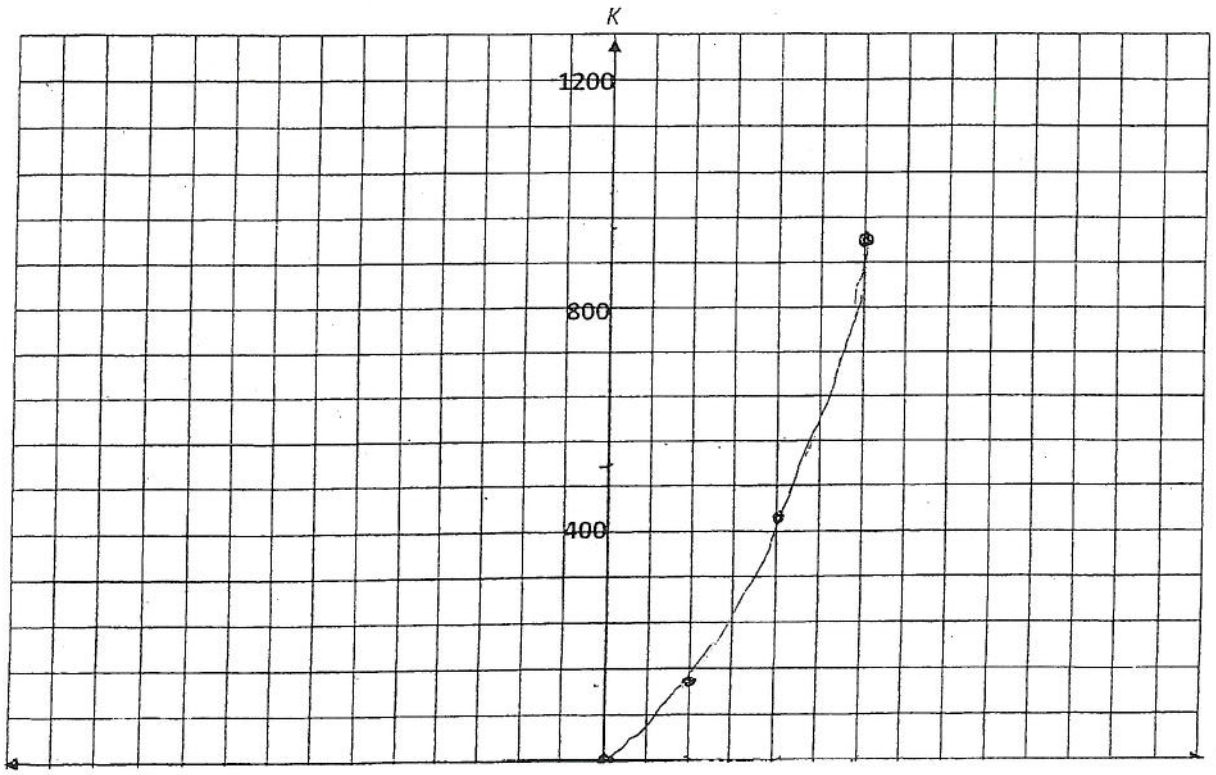
Complete the table to show Taryn's kinetic energy at the bottom of the ramp as a function of velocity.

Be sure to include at least 5 different velocities. $.5 \cdot 52 \cdot v^2 \rightarrow K = 26v^2$

Taryn's Energy

Velocity (m/s)	0	2	4	6	8
Kinetic Energy (Joules)	0	104	416	936	1664

1b. Graph Taryn's kinetic energy, K, as a function of velocity, v.



1c. Describe how the values in the table in 1a are represented in the graph in 1b.

In the graph, the values in the table are represented as points on the place where their velocity and Kinetic energy meet on the graph.

1d. Describe what one point on the graph means in the context of the situation.

One point on the graph shows Taryn's velocity and Kinetic energy at a certain point in time.

HomeLink G

□ On-demand Skatepark (A1.5.A, A1.2.B)

(A1.2.B)

2. Taryn needs to reach a velocity of 10 m/s at the bottom of the ramp to complete a trick. Kinetic energy at the bottom of the ramp equals potential energy at the top of the ramp. $(P = 9.8 \cdot m \cdot h)$ $P = 509.6h$

What is the minimum height, to the nearest tenth of a meter, on the ramp Taryn needs to start to reach a velocity of 10 m/s? $(K) 26v^2 = 509.6h (P)$

Show your work using words, numbers, and/or diagrams.

$$\frac{2600}{509.6} = \frac{509.6h}{509.6}$$

$$5.102040816 = h$$

$$h \approx 5$$

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(A1.5.A)

3. Jaden's mass is 60 kilograms. On any point of the ramp, Jaden's total energy is the sum of his potential energy and his kinetic energy. $K = \frac{1}{2}mv^2$ $30v^2$

Let E represent Jaden's total energy. $E = P + K$ $P = 9.8mh$

Write an equation to represent Jaden's total energy at any point on the ramp as a function of his height, h, and velocity, v.

Equation:

$E = 588h + 30v^2$

(A1.2.B)

4. During one trick, Jaden generated a total energy of 7,056 Joules.

- Determine all possible heights that could result in a total energy of 7,056 Joules.
- Determine all possible velocities that could result in a total energy of 7,056 Joules.

Show your work using words, numbers, and/or diagrams.

~~$\frac{1}{2}mv^2 + 9.8mh = E$~~ $h = 12$

$30v^2 + 588h = 7056$

aaaaaaah!

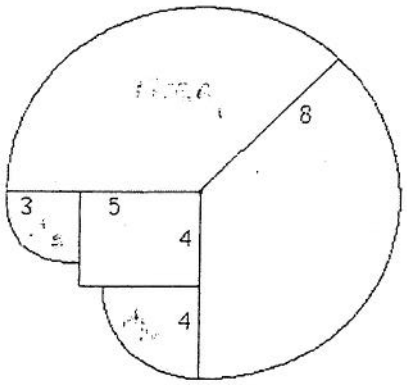
height $0 \leq h \leq 12$

velocity $0 \leq v \leq 15.3$

DRAFT

Goat Circles

A goat is tied to the corner of a 5-by-4-meter square shed by a 8-meter piece of rope. The goat grazes on the grass causing what looks like perfectly circular designs.The farmer is interested in just how much grass the goat has consumed since being tied up to her shed. The goat can walk around the far corners as far as the rope will allow. Along the top side, five meters of rope will be stretched along the side, leaving another three meters "in play"; along the right-hand side, four meters will have been used, leaving another four meters. (See the diagram below. All measurements in the drawing are in meters.)



(A1.7.D, A1.8.E)(M1.6.D, A1.8.E)

1a. Calculate the largest area the goat has available to graze in.

- Write the equations you used to solve the problem. List and define all variables.
- Show the steps you used to solve the equations. Be sure to identify the variables used in the equations

Equations used:
$Circle_A = \pi r^2$ $\quad$ $Shed_A = L \times w$ $C_A = \pi r^2$
Variables:
C_A = [illegible] $\quad$ $\pi = 3.14$ $\quad$ r = radius of circle $\quad$ L = length $\quad$ w = width

Inclusion Task

□ On-demand

Goat Circles

(A1.7.D, A1.8.C 1.8.E)
(M1.6.D, M1.8.C, M1.8.E)

Show your work here:

$\pi = 3.14$

$r = 8m$

$Circle_A = \pi r^2$

$Circle_A = (3.14)(8)^2$

$= 200.96 m^2$

$Sheet_A = L \times W$

$= 5 \times 4$

$= 20 m^2$

$200.96 - 20$

$180.96 m^2$ Largest area for grazing

1b. Calculate the smallest area the goat has available to graze in.

- Write the equations you used to solve the problem. List and define all variables.
- Show the steps you used to solve the equations. Be sure to identify the variables used in the equations.

Equations used:

$Circle_A = \pi r^2$

Variables:

$Circle_A$ = Area of circle, $\pi = 3.14$ r = radius.

Show your work here:

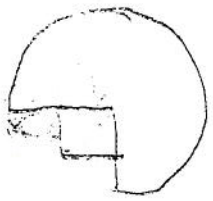
$\pi = 3.14$

$r = 3$

$Circle_A = \pi r^2$

$Circle_A = (3.14)(3^2)$

$Circle_A = 28.26 m^2$



Only use a quarter of the circle so divide by 4.

$\frac{Circle_A}{4} = 7.065 m^2$ Smallest area for goat.

Inclusion Task

Goat Circles

1c. Calculate the entire area the goat has available to graze in.

- Write the equations you used to solve the problem. List and define all variables.
- Show the steps you used to solve the equations. Be sure to identify the variables used in the equations.

Equations used:

Variables:

Show your work here:

$= 200.585\ m^2$

Inclusion Task

(A1.8.C)(M1.8.C)

2. The farmer decides she may no longer need the shed as very little is stored in it. If she takes the shed down she is trying to determine how much grazing area she would gain.

Calculate the amount grazing area gained by removing the shed. Determine the percentage gained from the original configuration if the shed is removed. Write a convincing argument for why or why not the shed should be removed.

- Write the equations you used to solve the problem. List and define all variables.
- Use numbers, drawings, equations in your written argument to the farmer.

Equations and Variables used:

$Shed_A = L \times W$

$Shed_A$ = area of shed

L = length

W = width

$Shed_A = 5 \times 4$

$= 20\ m^2$

$Circle = \pi r^2$ (of full circle)

$Circle_A = 3.14(8^2)$

$= 200.96\ m^2$

Total − shed = Gained

$200.96 - 20 = 180.585^2 = 375\ m^2$

Written Argument:

The farmer is using 200.585 m^2 now. If she removed the shed she would get to use 200.96. She would not gain that much by removing the shed. I think she should leave it so she has a place to put shade. She can have a place to put her goat if it got old.

Inclusion Task

Safe Driving

Different roads have different speed limits for various reasons. The primary reason is the relationship between the speed of a vehicle and the time it takes the vehicle to come to a full stop. Other factors that affect the stopping time include road conditions and braking efficiency.

Investigators gather information and make estimates at the scene of an accident to determine if vehicles were traveling at unsafe speeds. Information includes measuring the length of the tire's skid marks which represent the total stopping distance. Estimates are made for the drag factor caused by road conditions and the braking efficiency of the vehicles. Investigators can then estimate a vehicle's speed using mathematical equations.

One of these equations, $S = \sqrt{30Tfn}$, can be used to estimate the speed of the vehicle based on the length of the skid marks, the drag factor, and braking efficiency. In the equation:

S = the speed of the vehicle in miles per hour
T = the total stopping distance in feet based on the length of the skid marks
f = the drag factor
n = the braking efficiency

Another equation, $T = S + \frac{S^2}{20}$, can be used to estimate the total stopping time based on the speed of the vehicle. In the equation:

T = the total stopping distance in feet
S = the speed of the vehicle in miles per hour

Inclusion Task

(A1.7.D)

1a. Investigators often use a graph to quickly determine the expected length of the skid marks, which represent the total stopping distance, based on the speed of a vehicle based. This is difficult when the equations are not both solved for the same variable.

Solve the equation $S = \sqrt{30Tfn}$ for T.

Show the algebraic steps you took to solve the equation.

1. $S^2 = (\sqrt{30Tfn})^2$

2. $\frac{S^2}{(30)(f)n} = \frac{30Tfn}{(30)(f)n}$

$T = \frac{S^2}{(30)(f)n}$

(A1.1.C,A1.3.B)

2a. Investigators graph these equations to model common situations. One of these common situations is a 4-door vehicle stopping on dry pavement. Dry pavement has a drag factor of approximately 0.7 and the average 4-door vehicle has a braking efficiency of 80%.

One first step in creating such a graph is making a table of values.

Complete the tables of values for both equations when $f = 0.7$ and $n = 0.8$. Round all answers to the nearest tenth of a foot.

$S = \sqrt{30Tfn}$

S	T
20	23.81
40	95.24
60	214.28
80	380.95
100	595.24
120	857.14

$T = S + \frac{S^2}{20}$

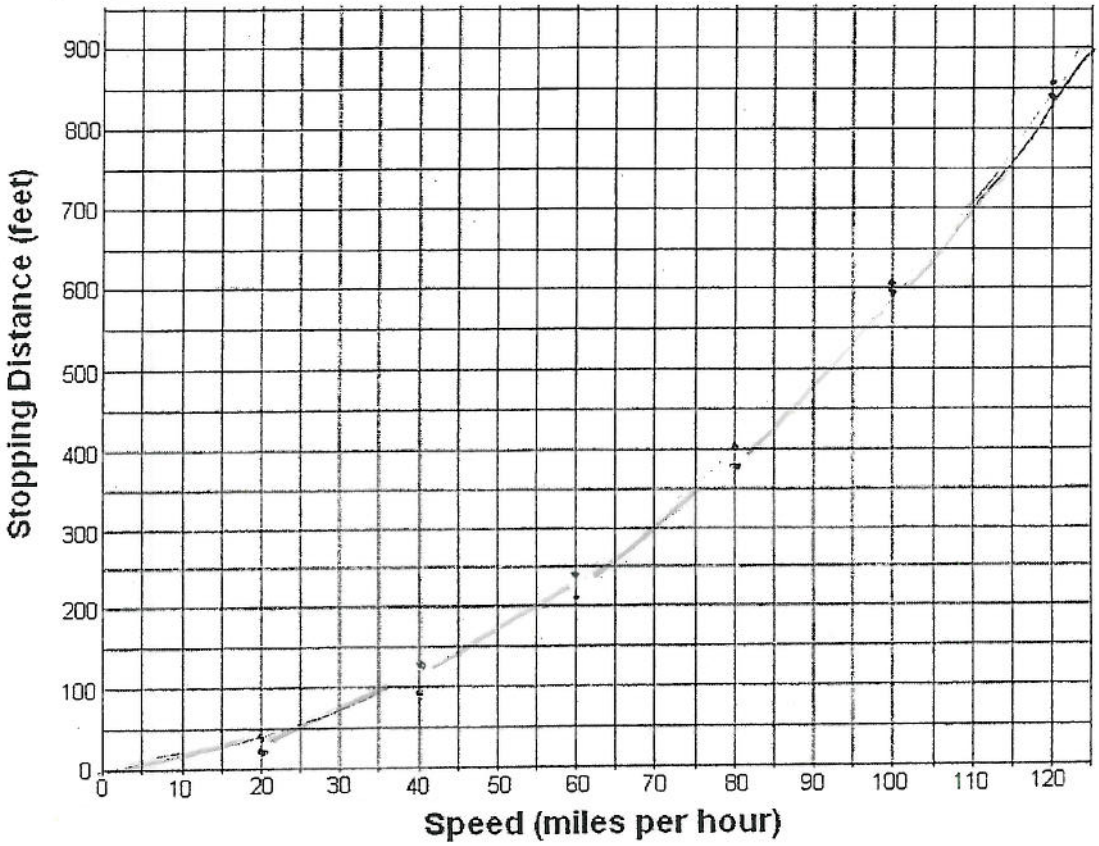
S	T
20	40
40	120
60	240
80	400
100	600
120	840

Inclusion Task

2b. A table, however, does not show as many values as a graph.

Graph both equations, $S = \sqrt{30Tfn}$ and $T = S + \frac{S^2}{20}$, for values of S between 0 and 120 to represent the stopping distance of a 4-door vehicle on dry pavement.

Speed vs. Stopping Distance of 4-door Vehicles on Dry Pavement



2c. Explain the advantages of both a table of data and corresponding graph when investigating an accident.

A table gives exact values for a given speed. A graph allows you to determine any value on the line. Each specific speed value corresponds to a stopping distance on the line.

Inclusion Task

(A1.1.D)

3a. The graph shows the general relationship between speed and stopping distance of a 4-door vehicle on dry pavement is very similar for both equations. However, there is one speed for which the stopping distance is the same for both equations.

Determine the exact speed (to the nearest tenth) of a 4-door vehicle on dry pavement that, for both equations, the stopping distance is exactly the same.

Show how you algebraically solved the problem.

$S = \sqrt{30(T)(f)n}$ $\quad T = S + \frac{S^2}{20}$ $\quad f = .7$

$T = \frac{S^2}{30(f)(n)}$ $\quad n = .8$

$\frac{S^2}{30(f)(n)} = S + \frac{S^2}{20} \longrightarrow \frac{S^2}{(30)(.7)(.8)} = S + \frac{S^2}{20}$

$\frac{S^2}{16.8} = S + \frac{S^2}{20}$

$S^2 = 16.8S + \frac{16.8}{20}S^2 \longrightarrow S^2 = 16.8S + .84S^2$

$-.84S^2 \qquad -.84S^2$

$\frac{0.16S^2}{S} = \frac{16.8S}{S}$

$\frac{.16S}{.16} = \frac{16.8}{.16}$

$S = 105$

Inclusion Task

3b. Officer Davis, an accident investigator, responded to a call of a car hitting a deer. After he arrived and found no one, including the deer, was seriously injured, he began his investigation. He measured the skid marks made by the vehicle, and they were approximately 290 feet long. The car must have been moving fast enough that 290 feet was not enough space to stop. Officer Davis estimated the drag factor to be 0.65 because there was a little rain on the road. He also estimated the braking efficiency of the car was 85%.

Use the equation, $S = \sqrt{30Tfn}$ to determine the speed the car was traveling and the equation, $T = S + \frac{S^2}{20}$, to determine the minimum total braking distance that would have been needed for the car to stop without hitting the deer, based on the speed it was traveling.

Show how you algebraically determined the minimum distance.

$f = 0.65$

$n = 85\% = 0.85$

$T = 290$ ft

$S = \sqrt{30(T)(f)n}$

$= \sqrt{30(290)(.65)(.85)}$

$S = 69.3$ miles/hr

$T = S + \frac{S^2}{20}$

$= 69.3 + \frac{69.3^2}{20}$

$= 309.64$ ft

Inclusion Task