STATE BOARD OF EDUCATION

HEARING TYPE: ___X___ ACTION

DATE: January 9-10, 2008

SUBJECT: **MATH STANDARDS REVIEW UPDATE, MATH THIRD CREDIT AND DATE OF EFFECTIVENESS FOR REQUIRING MATH WASL FOR GRADUATION**

SERVICE UNIT: Edie Harding, Executive Director
State Board of Education

PRESENTER: Steve Floyd, Board Lead
Edie Harding, Executive Director
Linda Plattner, CEO of Strategic Teaching

BACKGROUND:

Math Standards

The Board completed its review of the current K-12 math standards in September. OSPI had prepared a draft of the new math standards for public input, the standards will then be completed by January 31st. The Board will receive a brief update on OSPI's revised draft math standards.

The Third Math Credit and Math Content for Three Credits

During the last session, the legislature requested the State Board of Education to "revise high school graduation requirements to include a minimum of three credits of mathematics, one of which may be a career and technical course equivalent in mathematics, and prescribe the mathematics content in the three required credits." [1] The Board was asked to complete this work by December 1, 2007. This work has now been extended for adoption (by legislative agreement) at the March Board meeting, but the Board needs to give guidance to staff about how to proceed to draft a rule for the third credit of math.

There are three options to consider. All three options would incorporate a career and technical education option and appropriate accommodations for Special Education students). Linda has prepared pros and cons in her paper for Options 1 and 2. Staff is providing a third option to consider, which would combine Option 1 and 2. While the expected effective date for any of these options is intended to be for the Class of 2013, the Board may select a different phase in date.

Option 1:
The content in the third math credit would exceed the content taught in the first two years of high school. Courses, whether academic or CTE, that fit into this category would include some content from grades 9 and 10, but at least 50 percent of the content would go beyond grade 9 and 10 content. Mastery of that content would be expected.

Option 2:
The content in the third math credit would be the same content as is in Algebra 2. This doesn't mean that it would need to be a formal Algebra 2 course. For example, it could be a CTE business course in applied excel that required two years of enrollment to earn the one math credit.

Option 3:
The content in the third math credit would be the same content as is in Algebra 2, but a student and his/her family could meet with a high school counselor after the first year of high school and decide through a formal sign off on the high school and beyond plan that the student will take the math outlined in Option 1.

Based upon the Board's decision, staff will draft a rule by January 23rd for action at the March Board meeting.

Math WASL for Graduation

During the 2007 session, the legislature deferred the graduation requirement that students must meet the math standard on the 10th grade WASL until the class of 2013, but they also said that the Board could decide to move the requirement back to the Class of 2012. The Board will be asked at its January meeting to give staff guidance as to whether or not move the requirement of meeting the math standard on the 10th grade WASL to the Class of 2012 as a high school graduation requirement. If the Board decides in the affirmative, staff will draft a rule by January 23rd for action at the March Board meeting.

MATHEMATICS UPDATE

Math Standards

The Board has been examining math issues for over a year. Last fall the Board worked with the Office of the Superintendent of Public Instruction (OSPI) and the Professional Educator Standards Board to develop a Joint Math Action Plan to address the system issues for math in K-12, including topics such as aligning standards, curriculum and assessment, teacher supply and professional development. Last winter the Board hired Strategic Teaching to conduct an independent review of the K-12 math standards and to work with the Board's Math Panel. Those recommendations were reviewed at three focus groups and through online feedback forms. The recommendations were approved by the Board at its September 2007 meeting.

OSPI has hired the Dana Center to facilitate a process to rewrite the math standards based on those recommendations. The revisions, due to the legislature by January 31, 2008, were released in draft by OSPI on December 4. The Board's Math Panel met with Dr. Cathy Seeley from the Dana Center on December 13th to provide feedback on the revised standards. A copy of Seeley's PowerPoint is included in your packet. At the January meeting, Steve Floyd will share with you the discussion at the Math Panel meeting.

Strategic Teaching's Linda Plattner will review the final standards the first week in February and then meet with the Board's Math Panel on February 11th to determine to what extent OSPI has followed the Board's recommendations. This information will be shared with all of you when it is completed in February.

OSPI has made a lot of progress in a very short time. They are reaching out to many different groups to get feedback. Many groups are weighing in with comments, including our own math panel. The high school standards are currently in one block 9-12 with no breakout by grade level. This presents the Board with several challenges, including: What is expected for the first two credits of high school math and what should the third credit be?

While we expected the content to include Algebra I, Geometry and Algebra II, there are many standards on data, probability and statistics. To look at the standards, go to the home page of the OSPI Web site: http://www.k12.wa.us. The Dana Center is currently working on defining the standards by grade and course content, but it is not clear if this information will be available by the Board's January meeting.

The Third Math Credit and Math Content for Three Credits

At the November meeting the Board agreed that it made sense to ask for an extension on the Board's required adoption of a third math credit from December 1, 2007 to the end of March 2008. This was done for two reasons: The Board felt it wise to wait until they could see what had happened with the math standards rewrite for high school and the Board was in the middle of conducting its public outreach on math. The Board directed Edie Harding to draft a letter to that effect. Edie also met with the chairs and other legislators from the Senate and House education

committees and they agreed it made sense to wait until March. It is understood that this third credit would still go into effect for the class of 2012.

Linda Plattner was retained to assist the Board with a review of the third math credit and to explore ways that Career and Technical Education (CTE) courses could be used as equivalents as well as to draft suggested math content for the three credits. At the November Board meeting she shared (via phone) her PowerPoint to present some initial ideas. At the January meeting the Board will be asked to give staff guidance about the third math credit. There are three options to consider. All three options would incorporate a CTE option and appropriate accommodations for Special Education students. Linda has prepared pros and cons in her paper for Options 1 and 2. Staff is providing a third option to consider, which would combine Option 1 and 2. While the expected effective date for any of these options is intended to be for the Class of 2013, the Board may select a different phase in date.

Option 1:
The content in the third math credit would exceed the content taught in the first two years of high school. Courses, whether academic or CTE, that fit into this category would include some content from grades 9 and 10, but at least 50 percent of the content would go beyond grade 9 and 10 content. Mastery of that content would be expected.

Option 2:
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Option 3:
The content in the third math credit would be the same content as is in Algebra 2, but a student and his/her family could meet with a high school counselor after the first year of high school and decide through a formal sign off on the high school and beyond plan to allow the student to take the math outlined in Option 1. This third option is similar to how other states have addressed the Algebra 2 issue.

The biggest question will be whether or not to align the third credit with Algebra 2 for all students and to ensure a career and technical education equivalent. Linda Plattner has prepared the enclosed paper, which includes research on the impact of Algebra 2 on students as we heard during our public outreach sessions concerns about dropouts and what graduates really need today to be successful in careers and postsecondary education.

The work on the Meaningful High School diploma may consider other issues this winter and spring including: a fourth credit of math and or a requirement for students to take math in their senior year.

During the Board's fall public outreach sessions, people were asked what they thought about the third credit of math. In general, the majority of people supported a third credit of math but wanted different choices of math for students which did not include support requiring Algebra 2 for all students. More specific information on findings from the outreach will be handed out at the meeting.

We are providing information on what other states are doing. Currently 14 states will or plan to require Algebra 2 as a high school graduation requirement. However, in all but two states students could elect to opt out of college pathway and take another kind of math credit that was not Algebra 2.

We have also provided an interesting article from the November 2007 issue of Education Leadership on the use of algebra: "How Mathematics Counts," by Lynn Arthur Steen.

At the January meeting Board members will hear from students who are taking vocational programs that use math and science at the New Market Skills center as well as a panel of K-12 and community and technical college math experts who will talk about how they are approaching higher level math for students that traditionally struggle with math.

The Board will be asked to review the options posed and give staff guidance on how to proceed as we must prepare a draft rule by January 23rd to allow sufficient time to go through the code reviser process to prepare for adoption (although we can make modifications) at the March Board meeting.

Linda Plattner will continue to work on the content of the three math courses, which will be available at the March Board meeting. She will review the work of the Dana Center, Achieve, and the National Council of Math Teachers.

Date for Math WASL as High School Graduation Requirement

During the 2007 session, the legislature deferred the graduation requirement that students must meet the math standard on the 10th grade WASL until the class of 2013, but they also said that the Board could decide to move the requirement back to the Class of 2012. The Board will be asked at its January meeting to give staff guidance as to whether or not move the requirement of meeting the math standard on the 10th grade WASL to the Class of 2012 as a high school graduation requirement. If the Board decides in the affirmative, staff will draft a rule by January 23rd for action at the March Board meeting. The Board should ask OSPI and other education stakeholders for an indication of system readiness to determine if it is wise to move the deadline back. Currently, Board staff lack sufficient information to make a recommendation.

STATE REPRESENTATIVE
40th DISTRICT
DAVE QUALL

State of
Washington
House of
Representatives

EDUCATION
CHAIRMAN
AGRICULTURE &
NATURAL RESOURCES

December 10, 2007

STATE BOARD OF EDUCATION

DEC 11 2007

RECEIVED

Mary Jean Ryan, Chair
Washington State Board of Education
PO Box 47206
Olympia, WA 98504-7206

Dear Mary Jean,

Last session the Legislature asked that the State Board revise the high school graduation requirements to include a minimum of three credits of mathematics, one of which may be a career and technical course equivalent in mathematics, and prescribe the mathematics content in the three required credits. This was to be done by December 1, 2007.

Since that time, there have been a number of ongoing efforts related to the development of math standards in the state which will not be completed until the end of January 2008. You have asked that you be allowed to wait until your March 2008 meeting to make your decisions based on information still outstanding.

I support your request and thank you for all the efforts of the State Board on behalf of the students of Washington State.

Sincerely,

Dave Quall

DAVE QUALL
State Representative
40th District

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K-12 Mathematics Standards Revision
OSPI Next Steps
December 13, 2007
SBE Math Panel Meeting

1. **Formative Feedback Groups: December 2007**
   → **Input received during December will be integrated into the next draft of the standards document in January.**
   - Project web site (feedback form and email feedback) - http://www.utdanacenter.org/wamathrevision/
   - Formative groups – regional, by expertise, grade-specific, by affiliation, etc…
     - CARC + (CARC members, including OSPI mathematics specialists, ESD Math Coordinators, WSECC representation, Math Helping Corps Coordinators, Transition Math Project)
     - Washington Education Research Association (December 6, 2007)
     - Where's the Math (December 8, 2007)
     - State Board of Education Math Panel (December 13, 2007)
     - Superintendent's Advisory Committee (January 3, 2008)
     - OSPI Bilingual Education Advisory Committee (BEAC) and ELL Mathematics team (Dec. 20, 2007)
     - Business/Industry (Partnership for Learning, Business Roundtable)
     - Legislators and Legislative Committees
     - Other…

2. **Formal Focus Groups: January 21 -29, 2008 (specific timing to be determined)**
   → **These groups will provide input/comment on the next draft of the revised standards developed as a result of December input.**
   - WA TOTOM (Washington Teachers of Teachers of Mathematics)
   - Math Leadership Alliance Advisory – North Central ESD
   - PTSA Math/Science Group
   - Transition Math Project
   - OSPI Curriculum Advisory and Review Council

3. **Public Community Forums: January 2008 (dates and times to be determined)**
   - Spokane, Yakima, Seattle, Vancouver
4. **Present Revised Standards to Legislature: January 31, 2008**
5. **Develop "Roll-out" and Support Plan for New Standards: January 2008**
   - Rollout and training to begin in Spring 2008

K-12 Mathematics Standards Revision

Update to the Washington Math Panel

Cathy Seeley
Charles A. Dana Center, University of Texas
December 13, 2007

Role of the Dana Center

- Manage and facilitate the standards revision process to assure fidelity and alignment with the SBE Review and Recommendations report.
- Work with Washington educators, mathematicians and expert advisors to develop comprehensive drafts of the revised standards.

The Commitment

- This work will be generated by Washington educators, Washington mathematicians and Washington citizens.
- There must be as many opportunities and vehicles as possible for feedback and input from Washington educators, Washington mathematicians and other Washington citizens.
- The revised math standards will balance 1) Washington's unique strengths and needs with 2) expert advice from mathematicians and practicing educators and 3) conformity to national directions.
- The strengths of the current math GLEs will be preserved, while addressing the SBE recommendations.

The Reality

- The timeline is (nearly) impossible.
- The pressure and stress on all involved is significant.
- Collaboration, consensus, and reflection are more challenging to accomplish on this timeline.
- This is a ***Preliminary Draft***.
- Readers will find improvements to suggest.
- Those suggestions will not agree.
- The commitment from the Washington team members is extraordinary.
- The only way any standards will work is with a long-term, comprehensive program of implementation support.

Mathematics Standards Revision Process Team Structure

- Standards Revision Team (Washington educators and other stakeholders)
- Editorial Team (Washington and out-of-state experts)
- Articulation Team
  (Washington and out-of-state experts)
- Project Management Team (OSPI, Dana Center)
- *and* opportunities for public input/feedback

Format of the Preliminary Draft: Priorities (Paragraphs)

- Three to four content priorities per grade K-8 describing the most important mathematics for students to learn.
- Three to five content priorities in each of four strands describing the most important mathematics for three years of math in grades 9-12.
  (Alg/Number, Functions/Analysis, Geom/Meas, Probability/Statistics)
- Two additional process priorities describing important mathematical processes for each grade level
  - Reasoning/Problem Solving
  - Mathematical Communication (including representations, vocabulary, symbolism, definitions)

Format of the Preliminary Draft: Expectations (Statements)

- Specific statements of what students should learn (left-hand column).
- Elaborations, clarifications and examples (right-hand column)

Format of the Preliminary Draft: Supporting Ideas (K-8)

- A summary paragraph, identifying other important content to be addressed at this grade level.
- Specific student expectation statements (left-hand column)
- Elaborations, clarifications, examples (right-hand column)

In Support of the Preliminary Draft: Thread Documents

- Number
- Operations
- Geometry
- Measurement
- Algebra
- Data Analysis

SBE Recommendation #1:
'...fortify the content and **raise the rigor**'

- 3.2.a: Introduces fraction concepts at grade 3 rather than grade 4
- 4.3.c: Introduces the use of formulas for finding perimeter and area measurements in grade 4 rather than in current grade 5 GLE 1.2.5.
- 5.1.a, 5.1.c, 5.1.d, 5.1.e, and 5.1.f: Addition and subtraction of fractions applies to all fractions and mixed numbers and does not limit which numbers are used in denominators as in the current grade 5 GLE 1.1.6.

SBE Recommendation #2:

'...importance of all aspects of mathematics: mathematics **content**, including **standard algorithms**; **conceptual understanding**; and **application** of mathematical processes within the content.'

- 5.1.d: Use efficient algorithms, including standard algorithms, for addition and subtraction of fractions (proper and improper fractions), decimals (to hundredths), and mixed numbers.
- 2.1.b: Represent numbers to at least 1000 in different ways using physical models, pictures, graphs, written words, and numerals and translate from one representation to another.
- 7.3.a: Solve problems for a wide variety of proportional situations including those involving similarity, congruence, probability, percent increase, and percent decrease.

SBE Recommendation #3:

'Identify those topics that should be taught for extended periods at each grade and show how topics develop over grade levels.'

- Four to six priorities per grade level K-8
- Sixteen priorities for grades 9-12
- 'Threads' documents... other possibilities?

SBE Recommendation #4:

'Increase the clarity, specificity, and measurability...'

- 3.S.b: Round whole numbers up to 10,000 to the nearest ten, hundred, and thousand.
  (Includes rounding as a specific expectation rather than being grouped with estimation strategies as in current Grade 3 GLE 1.1.8; makes clear what numbers are to be addressed.)
- 4.3.e : Find the area of non-rectangular shapes that can be composed or decomposed into rectangles.
  (Specifies a structure for decomposing shapes into rectangles, not in the current grade 4 GLE 1.2.6.)

SBE Recommendation #5:

'Write EALRs that restructure [standards to]…reflect both the conceptual and procedural sides of mathematics.'

- Replace K-12 EALRs with grade-level priorities describing content (conceptual/procedural) and processes *(See Priority 6.1 and related Expectations)*

Note from SBE Recommendation #5:

- 'We also suggest collapsing the process strands into fewer EALRs. We like the idea of reducing the number of EALRs from four to two:
  - 1) Reasoning and problem solving and
  - 2) Communication.'

- The Preliminary K-12 Washington Math Standards: Priorities at each grade level include two priorities on mathematical processes (total of five to six priorities):
  - 1) Reasoning/Problem Solving and
  - 2) Communication.

SBE Recommendation #6:

'...easily used by most people.'

- Descriptive paragraphs allow readers to see what's important.
- Paragraphs help teachers focus instruction.
- A reasonable number of expectations allows teachers to organize and focus instruction.
- Avoiding extra levels (of organization) allows communication of the most important ideas without excessive repetition.
- Fewer pages per grade, with organization tighter
  (Ex: Gr 3 EALRs/GLEs: 10 full pages; Prelim. Gr 3 standards: 8 pages, including large-font paragraphs and white space; 5 EALRs/15 components/40 GLEs/152 bullets; 5 grade-specific priorities/34 expectations)

SBE Recommendation #7:

'Create expert Standards Revision Teams for each grade band … and collect feedback.'

- Knowledgeable, committed Standards Revision Teams (K-2, 3-5, 6-8, 9-12) representing diverse perspectives on mathematics, teaching and learning

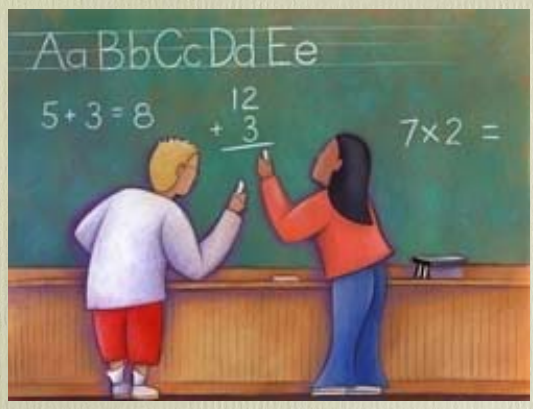
- Informal and formal focus groups, presentations, discussions, invited meetings, accessible website with online feedback, gathered daily, summarized and shared regularly with SRTs

Issues and Discussion Points

- Maintaining the integrity of the process, while addressing the SBE recommendations
- Maintaining the integrity of the process, even on a short timeline
- 'Understand'
- Priorities in descriptive paragraphs vs. student expectations in more specific terms
- Putting in perspective other states'/nations' standards and expert recommendations

Questions?

Thank you for your commitment to Washington teachers and students!



Background Paper for Third Math Credit Options
Linda Plattner, Strategic Teaching
December 2007

The State Board of Education has been tasked with revising Washington's high school graduation requirements to include a minimum of three credits of mathematics and to define the content in those credits. One of these credits can be a Career and Technology Education (CTE) credit.

There are three likely routes to earning the mathematics credits:

1. The traditional sequence of Algebra 1, Geometry, and Algebra 2.
2. Three years of integrated math.
3. The first two years of either of the above and one other course, which may be a CTE course.

The work of defining the content in each of the courses has begun.

There is a draft of the content for each course in the traditional sequence and these drafts will be finalized when Washington's new math standards are approved. In addition to the new math standards, the work of the National Math Advisory Panel,[1] Achieve's Traditional Plus Content[2], and feedback from the Washington Math Panel will be considered when the content for the traditional courses is finalized.

The content from the traditional courses will be used as the foundation of the content in the integrated courses. Generally, the content that is included in Algebra 1, Geometry, and Algebra 2 will be reorganized into the courses of Integrated Math 1, Integrated Math 2 and Integrated Math 3. Achieve's Integrated Math Course Sequence, the typical organization of content in integrated math programs, feedback from the Washington Math Panel, and the effect of the WASL will be considered when the content for the integrated math courses is finalized.

The third math credit might be either an academic course or a Career and Technical Education course. Because there are many possible courses that could serve as this third credit, it makes more sense to define the parameters of the content than to try to specify content for an indefinite number of courses. In

[1] On April 18, 2006, President Bush created the National Mathematics Advisory Panel. The panel is in the process of defining the content that should be included in Algebra.

[2] Achieve is an organization dedicated to raising expectations for all students. Thirty states, including Washington, are part of its coalition. Achieve has established high school exit standards and the content that should be included in each of the courses in the traditional and the integrated series.

other words, it makes more sense to describe the kind of content that is acceptable rather than to specifically define each topic.

The Board needs to make a decision about the grade level of content necessary for the third math credit. The draft of new math standards, which are not yet available by grade level or subject area, includes Algebra 2 content. It is assumed that Algebra 2 content will be included in the new standards as expectations for the third year of high school math. This seems to leave two viable choices for courses that would qualify as the third math credit:

Option 1:
The content in the third math credit would exceed the content taught in the first two years of high school. Courses, whether academic or CTE, that fit into this category would include some content from grades 9 and 10, but at least 50 percent of the content would go beyond grade 9 and 10 content. Mastery of that content would be expected.

While nothing is certain, the assumption is that the topics in grades 9 and 10 fit into Algebra 1 and Geometry and that grade 11 equates to Algebra 2.

This means that the third credit math course content could be some, but not all, of the topics associated with Algebra 2 or it could be an extension of grade 9 and 10 topics, such as a more sophisticated treatment of statistics and probability.

New academic or CTE courses will need to be created since few, if any, exist that meet these criteria. This aligns well with the work of CTE because the spring of 2008 marks the beginning of a 5-year initiative to develop Programs of Study.[3]

Option 2:
The content in the third math credit would be the same content as is in Algebra 2. This doesn't mean that it would need to be a formal Algebra 2 course. For example, it could be a CTE business course in applied excel that required two years of enrollment to earn the one math credit.

Option 1: The case against requiring Algebra 2 content

- Expecting all students to master Algebra 2 content will reduce the number of students who graduate from high school.

- Increasing the number of years students are required to take math is enough to ensure they will learn more mathematics, even if it is not Algebra 2.

[3] According to OSPI's CTE website, A program of study is "a planned program of courses and learning experiences that begins with exploration of career options, supports basic academic and life skills, and enables achievement of high academic standards, leadership, preparation for industry-defined work, and advanced and continuing education." Retrieved from http://www.k12.wa.us/CareerTechEd/

- The application of mathematics, particularly in CTE courses, adds a dimension of rigor that is as important as the increased sophistication of content in Algebra 2.

- Students have multiple opportunities without Algebra 2 including:
  - Acceptance into most state-approved apprenticeships;
  - Entry into 2-year community and technical colleges to pursue associate, certificate, or transfer programs; and
  - Participation in proprietary schools.

- Although it is true that students who have not taken Algebra 2 often begin college in non-credit bearing math courses, this has little effect on their graduation rates.[4] Nationally, 60% of students who start college with no remedial courses graduate, while 55% of students who take 1 remedial course graduate. The percentage of students who graduate drops with every additional remedial course that is taken, but the impact of remedial math courses is less profound than the impact of remedial courses in reading.

- While courses that go beyond the first two years of high school and yet are not equivalent to Algebra do not yet exist, this presents a wonderful opportunity. Courses could be developed that include rich and meaningful mathematics. Students not intending to pursue mathematics-intensive majors, should be able to select from a number of courses that meet their needs.

Option 2: Case for aligning to Algebra 2

- Washington graduates must compete nationally and internationally. A total of thirty-five states already require or are phasing in at least 3 years of math for graduation.[5]

- The skills and knowledge required to be college ready or to be qualified for a living-wage occupation are the same. ACT[6] found this to be the case when it compared the knowledge and skills in the "zone 3" category of WorkKeys to the knowledge and skills associated with college ready. WorkKeys, a widely used assessment system that matches job applicants and employees with high work-ready skills and skill needs, has 5 levels; Zone 3 was chosen for the comparison because it is the lowest level of the WorkKeys system that enables a worker to support a small family.

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[4] Adelman, Clifford. (Summer, 1998) "The kiss of death? An alternative view of college remediation." *National Crosstalk, 6(3).* Retrieved December 4, 2002, from http://www.highereducation.org/crosstalk

[5] Reys, B. J., et. al., (April, 2007) "High School Mathematics: State-Level Curriculum Standards and Graduation Requirements." *Center for the Study of Mathematics Curriculum.* Retrieved December 8, 2007 from mathcurriculumcenter.org/PDFS/HSreport.pdf

[6] ACT Issue Brief, 2006; *Ready for College and Ready for Work: Same or Different*?; Retrieved Dec. 10, 2007 from http://wwwhttp://www.act.org/path/policy/pdf/ReadinessBrief.pdf

The study found that the type of class in which the student gained the skills—academic or CTE—was not important. It was only important that the student be held to high expectations.

- Requiring Algebra 2 does not increase the drop out rate, especially if support is provided. At the very worst, some studies suggest that graduation rates would dip by about one percentage point or less. At best, such policies might actually help *improve* graduation rates[7]—especially if coupled with strong supports to help ninth graders pass algebra.

  Valerie Lee and David Burkam examined whether high schools that allow students to take more low-level math courses have higher graduation rates—again, all else being equal. Rather than low-level math helping to raise graduation rates, "for every two additional math courses offered *below* the level of algebra, students experienced more than a 30% *increase* in the odds of dropping out […] This finding flies in the face of those who say that high schools must offer a large number of undemanding courses to keep uncommitted students in school."[8]

  John Bishop and Ferran Mane looked across states to determine whether states that require students to complete more academic courses have higher dropout rates. They found that tougher graduation requirements have no statistically significant impact overall, and a slight negative impact for high-poverty students.[9]

- In Washington, a minimum of Algebra 2 is required for admittance to any 4-year college or university. The Washington Higher Education Coordinating Board set these requirements last year.

- In a pair of landmark studies that followed high school students through their postsecondary years, Clifford Adelman found that the highest level of math taken in high school has the most powerful relationship to earning a bachelor's degree. This is true regardless of student ethnicity, family income or parents' education levels. Students who complete Algebra 2 in high school *more than double* their chances of earning a four-year college degree. Those who do not take challenging math courses are more likely to end up in remedial courses and are more likely to drop out.[10]

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[7] Greene, J.P., (April, 2006) "Leaving Boys Behind: Public High School Graduation Rates," *Manhattan Institute*, Civic Report Mc 48.
[8] Lee, V. E. & Bukam, D. T. (2003). Dropping out of high school: The role of school organization and structure. American Educational Research Journal, 40(2), 353-393.
[9] Bishop, J. H., and Mane, F. (2004) "Educational Reform and Disadvantaged Students: are they better off or worse off?" *Center for Advanced Human Resource Studies: working paper series*
[10] Adelman, Clifford. *Answers in the Tool Box: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment,* Office of Educational Research and Improvement, U.S.

- High expectations, including Algebra 2, helps close the achievement gap. Taking a rigorous high school curriculum that includes math, at least through Algebra II, cuts in half the gap in college completion rates between white students and African American and Latino students.[11] In communities where a college-preparatory curriculum is not required, economically disadvantaged students are less likely to be in schools that offer college-prep courses, may not know which courses they need to take, may require approval of a guidance counselor or school administrator to enroll, or may be discouraged from choosing a rigorous course schedule.

The Kentucky example

Beginning in 2012, the State of Kentucky will implement an approach to the mathematics required for graduation that may be worth further investigation by SBE. In Kentucky, students will be required to:

- Enroll in a mathematics course every year of high school;
- Earn 3 credits of mathematics; and
- Learn the content in Algebra 1, Geometry, and Algebra 2.

One note-worthy aspect of Kentucky's system is that a variety of courses can be substituted for the traditional Algebra 1, Geometry, and Algebra 2 courses. Specifically, an integrated, applied, interdisciplinary, occupational, or technical course that prepares a student for a career path may be used, but only if the substituted course contains all of the core content.[12]

Another interesting aspect is that students must be enrolled in a math class every year, but only need 3 credits for graduation. This opens the door for CTE courses that require two years of participation to earn 1 math credit. Kentucky's system aligns well with the research that supports the importance of 4 years of math. Students who don't take math in their senior year lose valuable math skills that effect their placement in college level courses or skill level in other post-secondary options.

Department of Education, June 1999. Adelman, Clifford. *The Tool Box Revisited,* Office of Educational Research and Improvement, *U*.S. Department of Education, 2006.

[11] Adelman, Clifford. *The Tool Box Revisited,* Office of Educational Research and Improvement, *U*.S. Department of Education, 2006.

[12] Core content is the content in the standards and in courses that is "testable" on KERA, Kentucky's state assessment.

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Mathematics High School Graduation Requirements
50 States and District of Columbia
2008 and Beyond (*Updated December 21, 2007)*

State	Credits 2008	Credits 2009+	Alg I	Alg II	Geom	Notes
Alabama	4		X		X	
Alaska	2					
Arizona	2	3 2012				The course content for at least two of the mathematics credits shall include Number Sense and Operations; Data Analysis, Probability and Discrete Mathematics; Patterns, Algebra and Functions; Geometry and Measurement; and Structure and Logic in preparation for proficiency at the high school level on the AIMS test and shall be taken consecutively beginning with the ninth grade, unless a student meets these requirements prior to the ninth grade pursuant to this subsection. The third credit shall include significant mathematics content as determined by the local school district governing board or charter school. Courses successfully completed prior to the ninth grade that meet the high school mathematics credit requirements may be applied toward satisfying those requirements.
Arkansas	3	4 2009	X	See notes	X	Effective 2010, **smart core** becomes the default college and work readiness curriculum and includes 4 credits, with math in grades 11 or 12; Algebra II, and a 4th class more advanced than Algebra II. Students who take the **core** curriculum must take 4 credits, including Algebra I and Geometry.
California	2		X			At least one course or a combination of the two courses must meet or exceed the rigor of the content standards for Algebra I. Students who took Algebra I before grade 9 must still complete 2 credits of math while in grades 9-12.
Colorado	0					Only state requirement is in social studies.
Connecticut	3					
Delaware	3	4 2011	X 2011	X 2011	X 2011	
District of Columbia	3	4	X 2011		X 2011	Currently, elementary algebra is required. Students must complete 1 credit of Algebra I and/or a higher level course and must enroll in the course no later than grade 9.

Note: States that are in bold type have opt-out policies.

State	Credits 2008	Credits 2009+	Alg I	Alg II	Geom	Notes
Florida	3	3-4 2011	X			Florida offers 3 graduation programs: 24 credit; 3-year, 18-credit college prep; and 3-year 18-credit career prep. Effective 2011: **24 credit:** 4 credits, Algebra I or its equivalent, or a higher-level math course **18-credit college prep**: 3 credits, Algebra I or above chosen from the list of courses that qualify for state university admission **18-credit career prep**: 3 credits, Algebra I or its equivalent (Equivalent = Algebra I Honors, Algebra Ia and Ib; Applied Math I and II, Integrated Math I and II; Pre-AICE Math, Pacesetter Math I
Georgia	3-4	4 2012	X	See notes	See notes	Current requirements vary depending on whether a student is enrolled in a **college prep** or **tech /career prep** pathway. Students in **college prep** take 4 credits, including Algebra I, Geometry, and Algebra II; **tech/career prep** take 3, including Algebra I or its equivalent. Effective 2012, students must take 4 credits of math, including Mathematics I, II, and III or their equivalents.
Hawaii	3					
Idaho	2	3 2013	X 2013		X 2013	Classes tied to Algebra I and Geometry standards, including 1 credit in the senior year.
Illinois	2	3 2009	X 2010		X 2010 Geom. content	One course must "include Geometry content," effective 2010.
Indiana	2	3 2010	X See notes			Effective 2011, all students must earn a **Core 40** Diploma unless student qualifies to **opt out** for a **General Diploma**. Students in **Core 40** must take one of two course sequences: Algebra I, Geometry and Algebra II or Integrated Math I, II, III. Students are required to take a math or physics course during their junior or senior year. Students in **General Diploma** must take 1 credit in Algebra or Integrated Math I.
Iowa	0	3 2011				State is establishing requirements for first time in all subjects, effective 2011.
Kansas	2	3 2009				Courses including "algebraic and geometric concepts."
Kentucky	3		X	X 2012	X	An integrated, applied, interdisciplinary or technical/occupational course that prepares a student for a career path based on the student's Individual Learning Plan may be substituted for a traditional Algebra I, Geometry or Algebra II course. This

State	Credits 2008	Credits 2009+	Alg I	Alg II	Geom	Notes
						decision is made on an individual student basis. The course must meet the content standards in the program of studies. Pre-Algebra shall not be counted as one of the three required Mathematics credits for high school graduation but may be counted as an elective. Mathematics shall be taken each year of high school.
Louisiana	3	4	X 2009 See notes	X 2012 See notes		Algebra I or Integrated Math I. Effective 2009, students can earn an academic endorsement or a career/technical endorsement to the standard diploma but currently the math requirement is the same for each. Effective 2012, all students automatically will be enrolled in the Louisiana **Core 4** Curriculum, unless they opt out.
Maine	2					Students must achieve "standards of the system of learning results" in all eight content areas, effective 2010.
Maryland	3		X		X	
Massachusetts	0					Massachusetts has no state-mandated requirements. A recommended curriculum, **MassCore,** was approved by the Board in November 2007. **MassCore** recommends 4 credits of math, including completion of Algebra II or completion of the Integrated Math equivalent. All students are recommended to take a math course during their senior year.
Michigan	0	4 2011	X 2011	X 2011	X 2011	Michigan is establishing state requirements for the first time, effective for the class of 2011, when students must take the **Michigan Merit** curriculum. All students must take math in senior year. Per parental request and counselor approval, student may complete **personal curriculum** with modified math requirements, but only after student has completed 2.5 credits of math and if student completes 3.5 credits of math before graduation, including 1 credit during senior year. All modifications still require Algebra II, but in varying amounts (e.g., .5 credit instead of 1), over extended time (e.g., 2 years instead of 1), or in a career and technical education program.
Minnesota	3		See notes	X 2015	See notes	Currently, 3 credits include "algebra, geometry, statistics and probability content sufficient to satisfy the academic standards." Effective class of 2011: Students must complete Algebra I by end of grade 8 and pass the state test (MCA-II/GRAD) in math in grade 11. Effective 2015: Students must complete an "Algebra II

Mathematics Requirements, 2008 and Beyond: 50 States and District of Columbia
Source: Education Commission of the States Standard High School Graduation Requirements database (last updated March 2007), updated by the Washington State Board of Education, 2007

State	Credits 2008	Credits 2009+	Alg I	Alg II	Geom	Notes
						credit or its equivalent." A CTE course may fulfill a general science, math or arts credit requirement.
Mississippi	3	4 2009	X			Effective 2012, Mississippi will require all students to complete a college preparatory curriculum unless they **opt out.** Both options require 4 credits, but the college preparatory curriculum requires Algebra I and two higher courses; those students who opt out take Algebra I and one higher course.
Missouri	2	3 2010				
Montana	2					Vocational/technical education
Nebraska	0					No state requirements; all local
Nevada	3					
New Hampshire	2					
New Jersey	3					
New Mexico	3		X			
New York	3					
North Carolina	3-4		X	See notes	See notes	Depends on pathway; students in **career prep** must take 3 credits, including Algebra I. Those in **college technical prep** must take 3 credits, including Algebra I, II, Geometry; or Algebra I, Technical Math I & II, or Integrated Math I, II, III. Students in **college prep** pathway take 4 credits, including Algebra I, II, Geometry, (or Integrated Math I, II, III) and a higher level course for which Algebra II is a prerequisite.
North Dakota	0					No specific state requirement beyond total credits; all local.
Ohio	3	4 2014		X 2014		
Oklahoma	3		X	See notes	See notes	Depends on curriculum. Oklahoma has a **college preparatory/work ready curriculum**, but students may **opt out** for a **core curriculum**. Effective 2010, students in the **college preparatory/work ready curriculum** must choose courses from Algebra I, II, Geometry, Trigonometry, Math Analysis, Calculus, Advanced Placement Statistics or any mathematics course with content and/or rigor above Algebra I and approved for college admission requirements. Students in the **core curriculum** must take 3 credits of math, including 1 credit of Algebra I or Algebra I taught in a contextual methodology, and 2 credits chosen from a prescribed list including all of the above courses and adding Statistics and/or Probability; Computer Science I, II; Mathematics of Finance; Intermediate Algebra, and others.

State	Credits 2008	Credits 2009+	Alg I	Alg II	Geom	Notes
Oregon	2	3 2010	X 2014			Effective 2014, Algebra I and above.
Pennsylvania	0					No state requirements; all local.
Rhode Island	4					4th credit must be math-related, such as computer programming, physics or accounting.
South Carolina	4					
South Dakota	3		X	See notes	See notes	Effective class of 2010, **advanced program** includes Algebra I, II, and geometry. **Standard program** requires Algebra I. All students must complete **advanced program** unless excused by parent/guardian and school counselor or school administrator.
Tennessee	3		X See notes	See notes	See notes	Depends on pathway. Students in **university prep** programs must take 2 credits in Algebra II, Geometry or other advanced math course or 2 credits in Integrated math II and III. Class of 2009 must take one of the following: "Algebra II, Geometry, Integrated Math II, or Technical Geometry."
Texas	3	4 2011	X	See notes	X	Depends on program. **Recommended** program includes Algebra I, II, and Geometry. **Minimum** program requires Algebra I and Geometry.
Utah	2	3 2011	X		X	
Vermont	3					
Virginia	3		x			Algebra I and higher, including at least two course selections from among: Algebra I, Geometry, Algebra II, or other math courses above the level of algebra and geometry.
Washington	2					
West Virginia	3	4 2010	X	See notes	X	Depends on pathway. Recommended sequence for **professional** pathway is Algebra I, Geometry, Algebra II, Trigonometry, and Pre-Calculus; for **skilled** pathway: Algebra I, Geometry, conceptual mathematics, college transition mathematics, or Algebra II.
Wisconsin	2					
Wyoming	3					Depends on endorsement. **Comprehensive endorsement:** standard requirements plus proficient performance on common core of knowledge and skills in math. **General endorsement:** proficient performance in a majority of nine subject areas, which include math.

Mathematics Requirements, 2008 and Beyond: 50 States and District of Columbia
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For the Success of Each Learner

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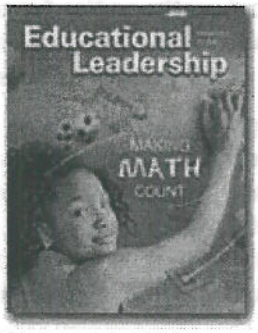
How Mathematics Counts



November 2007

Lynn Arthur Steen

Fractions and algebra represent the most subtle, powerful, and mind-twisting elements of school mathematics. But how can we teach them so students understand?

Much to the surprise of those who care about such things, mathematics has become the 600-pound gorilla in U.S. schools. High-stakes testing has forced schools to push aside subjects like history, science, music, and art in a scramble to avoid the embarrassing consequences of not making "adequate yearly progress" in mathematics. Reverberations of the math wars of the 1990s roil parents and teachers as they seek firm footing in today's turbulent debates about mathematics education.

Much contention occurs near the ends of elementary and secondary education, where students encounter topics that many find difficult and some find incomprehensible. In earlier decades, schools simply left students in the latter category behind. Today, that option is neither politically nor legally acceptable. Two topics—fractions and algebra, especially Algebra II—are particularly troublesome. Many adults, including some teachers, live their entire lives flummoxed by problems requiring any but the simplest of fractions or algebraic formulas. It is easy to see why these topics are especially nettlesome in today's school environment. They are exemplars of why mathematics counts and why the subject is so controversial.

Confounded by Fractions

What is the approximate value, to the nearest whole number, of the sum 19/20 + 23/25? Given the choices of 1, 2, 42, or 45 on an international test, more than half of U.S. 8th graders chose 42 or 45. Those responses are akin to decoding and pronouncing the word *elephant* but having no idea what animal the word represents. These students had no idea that 19/20 is a number close to 1, as is 23/25.

Neither, it is likely, did their parents. Few adults understand fractions well enough to use them fluently. Because people avoid fractions in their own lives, some question why schools (and now entire states) should insist that all students know, for instance, how to add uncommon combinations like 2/7 + 9/13 or how to divide 1 3/4 by 2/3. When, skeptics ask, is the last time any typical adult encountered problems of this sort? Even mathematics teachers have a hard time imagining authentic problems that require these exotic calculations (Ma, 1999).

Moreover, many people cannot properly express in correct English the fractions and proportions that *do* commonly occur, for instance, in ordinary tables of data. A simple example illustrates this difficulty (Schield, 2002). Even though most people know that 20 percent means 1/5 of something, many cannot figure out what the something is when confronted with an actual

example, such as the table in Figure 1. Although calculators can help the innumerate cope with such exotica as 2/7 + 9/13 and 1 3/4 ÷ 2/3, they are of no help to someone who has trouble reading tables and expressing those relationships in clear English.

Figure 1. The Challenge of Expressing Numerical Data in Ordinary Language

Not available for electronic dissemination.

These examples illustrate two very different aspects of mathematics that apply throughout the discipline. On the one hand is calculation; on the other, interpretation. The one reasons *with* numbers to produce an answer; the other reasons *about* numbers to produce understanding. Generally, school mathematics focuses on the former, natural and social sciences on the latter. For lots of reasons—psychological, pedagogical, logical, motivational—students will learn best when teachers combine these two approaches.

There may be good reasons that so many children and adults have difficulty with fractions. It turns out that even mathematicians cannot agree on a single proper definition. One camp argues that fractions are just names for certain points on the number line (Wu, 2005), whereas others say that it's better to think of them as multiples of basic unit fractions such as 1/3, 1/4, and 1/5 (Tucker, 2006). Textbooks for prospective elementary school teachers exhibit an even broader and more confusing array of approaches (McCrory, 2006).

Instead of beginning with formal definitions, when ordinary people speak of fractions they tend to emphasize contextual meaning. Fractions (like all numbers) are human constructs that arise in particular social and scientific contexts. They represent the magnitude of social problems (for example, the percentage of drug addiction in a given population); the strength of public opinion (for example, the percentage of the population that supports school vouchers); and the consequences of government policies (for example, the unemployment rate). Every number is the product of human activity and is selected to serve human purposes (Best, 2001, 2007).

Fractions, ratios, proportions, and other numbers convey quantity; words convey meaning. For mathematics to make sense to students as something other than a purely mental exercise, teachers need to focus on the interplay of numbers and words, especially on expressing quantitative relationships in meaningful sentences. For users of mathematics, calculation takes a backseat to meaning. And to make mathematics meaningful, the three *R*s must be well blended in each student's mind.

Algebra for All?

Conventional wisdom holds that in Thomas Friedman's metaphorically flat world, all students, no matter their talents or proclivities, should leave high school prepared for both college and high-tech work (American Diploma Project, 2004). This implies, for example, that all students should master Algebra II, a course originally designed as an elective for the mathematically inclined. Indeed, more than half of U.S. states now require Algebra II for almost all high school graduates (Zinth, 2006).

Advocates of algebra advance several arguments for this dramatic change in education policy:

- Workforce projections suggest a growing shortage of U.S. citizens having the kinds of technical skills that build on such courses as Algebra II (Committee on Science, Engineering, and Public Policy, 2007).
- Employment and education data show that Algebra II is a "threshold course" for high-paying jobs. In particular, five in six young people in the top quarter of the income distribution have completed Algebra II (Carnevale & Desrochers, 2003).
- Algebra II is a prerequisite for College Algebra, the mathematics course most commonly required for postsecondary degrees. Virtually all college students who have not taken

Algebra II will need to take remedial mathematics.

- Students most likely to opt out of algebra when it is not required are those whose parents are least engaged in their children's education. The result is an education system that magnifies inequities and perpetuates socioeconomic differences from one generation to the next (Haycock, 2007).

Skeptics of Algebra II requirements note that other areas of mathematics, such as data analysis, statistics, and probability, are in equally short supply among high school graduates and are generally more useful for employment and daily life. They point out that the historic association of Algebra II with economic success may say more about common causes (for example, family background and peer support) than about the usefulness of Algebra II skills. And they note that many students who complete Algebra II also wind up taking remedial mathematics in college.

Indeed, difficulties quickly surfaced as soon as schools tried to implement this new agenda for mathematics education. Shortly after standards, courses, and tests were developed to enforce a protocol of "Algebra II for all," it became clear that many schools were unable to achieve this goal. The reasons included, in varying degrees, inadequacies in preparation, funding, motivation, ability, and instructional quality. The result has been a proliferation of "fake" mathematics courses and lowered proficiency standards that enable districts and states to pay lip service to this goal without making the extraordinary investment of resources required to actually accomplish it (Noddings, 2007).

Several strands of evidence question the unarticulated assumption that additional instruction in algebra would necessarily yield increased learning. Although this may be true in some subjects, it is far less clear for subjects such as Algebra II that are beset by student indifference, teacher shortages, and unclear purpose. For many of the reasons given, enrollments in Algebra II have approximately doubled during the last two decades (National Center for Education Statistics [NCES], 2005a). Yet during that same period, college enrollments in remedial mathematics and mathematics scores on the 12th grade National Assessment of Educational Progress (NAEP) have hardly changed at all (NCES, 2005b; Lutzer, Maxwell, & Rodi, 2007). Something is clearly wrong.

Although we cannot conduct a randomized controlled study of school mathematics, with some students receiving a treatment and others a placebo, we can examine the effects of the current curriculum on those who go through it. Here we find more disturbing evidence:

- One in three students who enter 9th grade fails to graduate with his or her class, leaving the United States with the highest secondary school dropout rate among industrialized nations (Barton, 2005). Moreover, approximately half of all blacks, Hispanics, and American Indians fail to graduate with their class (Swanson, 2004). Although mathematics is not uniquely to blame for this shameful record, it is the academic subject that students most often fail.
- One in three students who enter college must remediate major parts of high school mathematics as a prerequisite to taking such courses as College Algebra or Elementary Statistics (Greene & Winters, 2005).
- In one study of student writing, one in three students at a highly selective college failed to use any quantitative reasoning when writing about subjects in which quantitative evidence should have played a central role (Lutsky, 2006).
- College students in the natural and social sciences consistently have trouble expressing in precise English the meaning of data presented in tables or graphs (Schield, 2006).

One explanation for these discouraging results is that the trajectory of school mathematics moves from the concrete and functional (for example, measuring and counting) in lower grades to the abstract and apparently nonfunctional (for example, factoring and simplifying) in high school. As many observers have noted ruefully, high school mathematics is the ultimate exercise in deferred

gratification. Its payoff comes years later, and then only for the minority who struggle through it.

In the past, schools offered this abstract and ultimately powerful mainstream mathematics curriculum to approximately half their students—those headed for college—and little if anything worthwhile to the other half. The conviction that has emerged in the last two decades that all students should be offered useful and powerful mathematics is long overdue. However, it is not yet clear whether the best option for all is the historic algebra-based mainstream that is animated primarily by the power of increasing abstraction.

Mastering Mathematics

Fractions and algebra may be among the most difficult parts of school mathematics, but they are not the only areas to cause students trouble. Experience shows that many students fail to master important mathematical topics. What's missing from traditional instruction is sufficient emphasis on three important ingredients: communication, connections, and contexts.

Communication

Colleges expect students to communicate effectively with people from different backgrounds and with different expertise and to synthesize skills from multiple areas. Employers seek the same things. They emphasize that formal knowledge is not, by itself, sufficient to deal with today's challenges. Instead of looking primarily for technical skills, today's business leaders talk more about teamwork and adaptability. Interviewers examine candidates' ability to synthesize information, make sound assumptions, capitalize on ambiguity, and explain their reasoning. They seek graduates who can interpret data as well as calculate with it and who can communicate effectively about quantitative topics (Taylor, 2007).

To meet these demands of college and work, K–12 students need extensive practice expressing verbally the quantitative meanings of both problems and solutions. They need to be able to write fluently in complete sentences and coherent paragraphs; to explain the meaning of data, tables, graphs, and formulas; and to express the relationships among these different representations. For example, science students could use data on global warming to write a letter to the editor about carbon taxes; civics students could use data from a recent election to write op-ed columns advocating for or against an alternative voting system; economics students could examine tables of data concerning the national debt and write letters to their representatives about limiting the debt being transferred to the next generation.

We used to believe that if mathematics teachers taught students how to calculate and English teachers taught students how to write, then students would naturally blend these skills to write clearly about quantitative ideas. Data and years of frustrating experience show just how naïve this belief is. If we want students to be able to communicate mathematically, we need to ensure that they both practice this skill in mathematics class and regularly use quantitative arguments in subjects where writing is taught and critiqued.

Connections

One reason that students think mathematics is useless is that the only people they see who use it are mathematics teachers. Unless teachers of all subjects—both academic and vocational—use mathematics regularly and significantly in their courses, students will treat mathematics teachers' exhortations about its usefulness as self-serving rhetoric.

To make mathematics count in the eyes of students, schools need to make mathematics pervasive, as writing now is. This can best be done by cross-disciplinary planning built on a commitment from teachers and administrators to make the goal of numeracy as important as literacy. Virtually every subject taught in school is amenable to some use of quantitative or logical arguments that tie evidence to conclusions. Measurement and calculation are part of all vocational subjects; tables, data, and graphs abound in the social and natural sciences; business

requires financial mathematics; equations are common in economics and chemistry; logical inference is fundamental to history and civics. If each content-area teacher identifies just a few units where quantitative thinking can enhance understanding, students will get the message.

The example of many otherwise well-prepared college students refraining from using even simple quantitative reasoning to buttress their arguments shows that students in high school need much more practice using the mathematical resources introduced in the elementary and middle grades. Much of this practice should take place across the curriculum. Mathematics is too important to leave to mathematics teachers alone.

Contexts

One of the common criticisms of school mathematics is that it focuses too narrowly on procedures (algorithms) at the expense of understanding. This is a special problem in relation to fractions and algebra because both represent a level of abstraction that is significantly higher than simple integer arithmetic. Without reliable contexts to anchor meaning, many students see only a meaningless cloud of abstract symbols.

As the level of abstraction increases, algorithms proliferate and their links to meaning fade. Why do you invert and multiply? Why is $(a + b)^2 \neq a^2 + b^2$? The reasons are obvious if you understand what the symbols mean, but they are mysterious if you do not. Understandably, this apparent disjuncture of procedures from meaning leaves many students thoroughly confused. The recent increase in standardized testing has aggravated this problem because even those teachers who want to avoid this trap find that they cannot. So long as procedures predominate on high-stakes tests, procedures will preoccupy both teachers and students.

There is, however, an alternative to meaningless abstraction. Most applications of mathematical reasoning in daily life and typical jobs involve sophisticated thinking with elementary skills (for example, arithmetic, percentages, ratios), whereas the mainstream of mathematics in high school (algebra, geometry, trigonometry) introduces students to increasingly abstract concepts that are then illustrated with oversimplified template exercises (for example, trains meeting in the night). By enriching this diet of simple abstract problems with sophisticated realistic problems that require only simple skills, teachers can help students see that mathematics is really helpful for understanding things they care about (Steen, 2001). Global warming, college tuition, and gas prices are examples of data-rich topics that interest students but that can also challenge them with surprising complications. Such a focus can also help combat student boredom, a primary cause of dropping out of school (Bridgeland, DiIulio, & Morison, 2006).

Most important, the pedagogical activity of connecting meaning to numbers needs to take place in authentic contexts, such as in history, geography, economics, or biology—wherever things are counted, measured, inferred, or analyzed. Contexts in which mathematical reasoning is used are best introduced in natural situations across the curriculum. Otherwise, despite mathematics teachers' best efforts, students will see mathematics as something that is useful only in mathematics class. The best way to make mathematics count in the eyes of students is for them to see their teachers using it widely in many different contexts.

My "Aha!" Moment

Douglas Hofstadter, Distinguished Professor of Cognitive Science, Indiana University, Bloomington.

I first realized the deep lure of mathematics when, at about age 3, I thought up

the "great idea" of generalizing the concept of 2 × 2 to what seemed to me to be the inconceivably fancier concept of 3 × 3 × 3. My inspiration was that since 2 × 2 uses the concept of two-ness *twice*, I wanted to use the concept of three-ness *thrice*! It wasn't finding out the actual value of this expression (27, obviously) that thrilled me—it was the idea of the fluid conceptual structures that I could play with in my imagination that turned me on to math at that early age.

Another "aha" moment came a few years later, when I noticed that $3^2 \times 5^2$ is equal to $(3 \times 5)^2$. Once again I was playing around with structures, not trying to prove anything. (I didn't even know that proofs existed!) It thrilled me to discover this pattern, which of course I verified for other values and found mystically exciting.

I believe that teachers should encourage playfulness with mathematical concepts and should encourage the discoveries of patterns of whatever sort. Any time a child recognizes an unexpected pattern, it may evoke a sense of wonder.

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